

Skeptical review: 3.2.2. Unsupervised flow regime classification

Summary

This manuscript presents an ambitious and wide-ranging post-processing analysis of a single high-resolution 3D radiation–hydrodynamic snapshot of a red-supergiant (RSG) donor in a binary, aiming to characterize *instantaneous* mass loss / mass-transfer morphology and driving. The authors map the spherical simulation output onto a Cartesian representation and analyze several masked regions (surface shell, L1 vicinity, and a threshold-defined “accretion stream” region) using PDFs and higher moments, two-point statistics and spectra, Q-criterion vortices, k-means clustering, streamline tracing, and local force-balance diagnostics (Sec. 2–3). The snapshot exhibits strongly intermittent, clumpy density and mass-flux fields with non-Gaussian PDFs and high kurtosis (Sec. 3.1), and the streamline visualization suggests broad plume-like outflows rather than a classical steady Roche-lobe overflow (RLOF) stream crossing L1 at this instant (Sec. 3.3). Force-ratio maps indicate radiation forces are negligible near the dense surface but become important in low-density outflows (Sec. 3.4). The work is conceptually interesting as a “snapshot benchmark”, but the strongest interpretive claims (e.g., challenging the RLOF paradigm, radiation as the dominant accelerator) currently rest on a single snapshot and on analysis choices (region thresholds, J- vs v-streamlines, magnitude-only force ratios) that require clearer specification, robustness tests, and more careful framing. Strengthening the simulation description, adding independent Eulerian flux diagnostics near L1, tightening coordinate/derivative definitions, and explicitly quantifying uncertainties would substantially improve reproducibility and the credibility of the headline conclusions.

Strengths

- Addresses an important and difficult problem: inherently 3D, time-dependent mass loss/mass transfer from RSG donors in binaries, beyond idealized steady RLOF (Sec. 1, Sec. 4).
- Employs a rich and modern diagnostic toolkit (PDFs/moments, correlations/structure functions, spectra, Q-criterion, clustering, streamlines, force-balance and radiative terms), potentially valuable as a reference workflow for future 3D RHD studies (Sec. 2–3).
- Presents clear evidence that the instantaneous density and mass-flux fields are highly intermittent and heavy-tailed, consistent with turbulent convection and clumpy outflows (Sec. 3.1; Table 1; Figs. 3–5).
- The instantaneous morphology highlighted in Sec. 3.3 (plume-like outflow; lack of a visually coherent L1 stream under the authors’ streamline procedure) is intriguing and worth documenting carefully as part of a time-variable picture.
- The radiation–hydrodynamic comparison (Sec. 2.5, Sec. 3.4; Table 2; Figs. 10–11) usefully emphasizes that radiative terms can become large once material is sufficiently dilute, motivating further time-dependent investigation.

Major issues

1. **Scope/inference is too broad for a single-snapshot study: the manuscript sometimes generalizes from one instantaneous state to global claims about RSG binary mass transfer (e.g., “challenges the traditional RLOF paradigm”), even though convection, pulsation, and orbital-phase effects can strongly modulate morphology (Abstract; Sec. 1; Sec. 3.3.1; Sec. 4.2–4.3).**

Recommendation: Systematically localize claims to “this snapshot in this simulation” throughout the Abstract, Sec. 1, Sec. 3.3.1, and Sec. 4.2–4.3. Add explicit time context: snapshot time relative to the orbital period and an estimated convective turnover time, and whether the simulation had reached a statistically steady state. If feasible, analyze even a small ensemble of additional snapshots (e.g., 5–10 across phase/time) to test persistence of (i) the “no L1-crossing streamlines” result and (ii) the force-balance/radiation trends; otherwise, add a clearly titled limitations subsection in Sec. 4.3 emphasizing that morphology and diagnostics may be transient.

2. **The underlying simulation setup and physics are insufficiently described for readers to judge realism or reproduce results (Sec. 2–2.1 focus on post-processing): code/solver, grid/domain, resolution, boundary conditions, EOS, opacities, radiation closure (FLD/M1, number of groups), binary parameters and frame, run duration, and snapshot selection criteria are unclear.**

Recommendation: Add a dedicated “Simulation setup” subsection early in Sec. 2 that concisely lists: numerical code and RHD method; grid geometry, resolution, and domain extents; boundary conditions; EOS and γ ; opacity treatment; radiation closure and groups; gravity (masses, separation, Roche potential) and whether the equations are solved in an inertial or corotating frame; and the snapshot time and selection rationale. Clearly state unit conventions and any mapping to physical units to avoid mixing code radii with ‘solar radii’ (Sec. 2.1–2.1.2). A compact parameter table (masses, separation $rm2$, ω , etc.) would help.

3. **The headline “no L1 stream” conclusion depends heavily on streamline tracing details that are under-specified and may yield false negatives. Additionally, streamlines are traced for the mass-flux field $J = \rho v$, which is not equivalent to Lagrangian trajectories (Sec. 2.4; Sec. 3.3.1–3.3.2). The crossing diagnostic (0/10,000 cross $x = x(L1)$) depends on seeding strategy, integration tolerances/termination, and the choice of a plane rather than a Roche/L1 nozzle surface.**

Recommendation: Expand Sec. 2.4 to specify the streamline integrator (e.g., RK order, step-size control, tolerances), interpolation scheme, maximum integration length/time, termination criteria, boundary handling, and a precise crossing criterion. Justify tracing J rather than v , and add a side-by-side test tracing v for a representative subset to show whether the qualitative conclusion changes. Strengthen the L1 claim with at least one independent Eulerian diagnostic: compute the instantaneous surface integral $\dot{M} = \iint J \cdot n \, dA$ through (i) a plane at $x(L1)$, and ideally also (ii) a small control surface around L1 (sphere/cylinder) or an approximate Roche-lobe

boundary. Improve seeding robustness by (a) seeding weighted by outward flux $\max(J_r, 0)$ on the surface shell and (b) increasing seed density near the substellar point toward the companion. If the result remains “no significant L1 flux in this snapshot”, state it as such rather than as a generic absence of RLOF.

4. **Region definitions—especially the heuristic “accretion stream” region—risk circularity and sensitivity to ad hoc thresholds** ($x1v > 684$, $vel1 > 0.1$, $\rho > 1e-4$), yet are used to support physical conclusions about outflow/radiation dominance (Sec. 2.1.2; Sec. 3.1.1–3.1.2; Sec. 3.4.1). Similar sensitivity may apply to the surface shell (650–684) and the “L1 vicinity” cube size.

Recommendation: In Sec. 2.1.2, motivate each threshold with physical scales (e.g., fractions of sound speed/escape speed, density relative to a surface/optical-depth criterion). In Sec. 3.1 and Sec. 3.4, provide sensitivity tests varying: (i) surface-shell radius/thickness, (ii) the L1 cube size/location, and (iii) the outflow thresholds, and report how key outputs (Table 1 moments; Table 2 force ratios) change. Consider renaming the “accretion stream” region to a neutral descriptor (e.g., “high-velocity outflow selection”) unless you show it feeds the companion. Add an “unbound/escaping” diagnostic where possible (e.g., Bernoulli parameter or v compared to local escape speed) to demonstrate that selected material plausibly escapes rather than being a transient eddy.

5. **Coordinate-basis consistency is unclear for vector quantities and is critical for streamlines, fluxes, and $|J|$: the paper uses spherical components (vel1–vel3) while later invoking Cartesian components (J_x, J_y, J_z) and Cartesian geometry ($x = x(L1)$ plane), without explicitly defining the conversions and basis assumptions (Sec. 2.2.1; Sec. 2.4; figures/tables using $|J|/J_{\text{mag}}$).**

Recommendation: Explicitly state whether streamline integration and flux calculations are performed in Cartesian (x, y, z) or spherical (r, θ, ϕ) coordinates. If Cartesian, provide the explicit conversion from (vel1, vel2, vel3) (and thus J) to (v_x, v_y, v_z) using the local spherical basis vectors; likewise define J_x, J_y, J_z only after conversion. If spherical, write the ODEs for (r, θ, ϕ) streamlines and explain the mapping to plotted (x, y, z). Fix the mass-flux magnitude definition so it matches the component basis used (e.g., $\sqrt{J_r^2 + J_\theta^2 + J_\phi^2}$ vs $\sqrt{J_x^2 + J_y^2 + J_z^2}$).

6. **Force-balance and “radiation-dominated acceleration” interpretation currently relies mainly on ratios of force magnitudes from instantaneous fields, with unclear treatment of rotating-frame/inertial terms and limited connection to acceleration along outflow direction or to escape/unbinding (Sec. 2.5.1–2.5.2; Sec. 3.4.1; Sec. 4.2–4.3). Computing $\nabla \cdot P_r$ in curvilinear coordinates is also easy to implement incorrectly, yet formulas/validation are not given.**

Recommendation: In Sec. 2.5.1, specify the frame and the exact momentum-equation decomposition you are comparing (inertial vs corotating). If corotating, include centrifugal and clarify treatment of Coriolis (not representable as $\nabla\Phi$). Provide explicit component formulas for $-\nabla P_g$, $-\rho\nabla\Phi$ (or the effective Roche potential), and $\nabla(\nabla\cdot P_r)_i$ in the coordinate basis actually used, including geometric terms. In Sec. 3.4.1, complement magnitude ratios with directional diagnostics: project accelerations along \hat{v} (or streamline tangent), e.g., a_{\parallel} , to avoid misleading “dominance” when forces are orthogonal. Add a minimal consistency/validation check (e.g., compare net force density $\rho(\nabla\cdot P_r)$ where appropriate; or, if possible, compare to dv/dt from nearby snapshots). Temper language about radiation “unbinding” material unless you provide an order-of-magnitude integration along a path and compare to escape velocity. = $\nabla(\nabla\cdot P_r)\cdot\hat{v}/\rho$ vs $a_{\text{grav},\parallel}$ and $a_{\text{gas},\parallel}$

7. **High-order statistics (skewness/kurtosis) and heavy-tail claims may be numerically fragile without clarifying weighting, sample sizes, and uncertainty. On a spherical grid, cell volumes differ; treating each cell equally can bias PDFs/moments. Tail moments are also sensitive to outliers, masking, and binning choices (Sec. 2.2.2–2.2.3; Sec. 3.1; Table 1; Figs. 3–5).**

Recommendation: Report, per region, the number of cells and whether statistics/PDFs are volume-weighted, mass-weighted, or unweighted; justify the choice. Specify PDF construction (bin counts/ranges, log vs linear variables, normalization, handling of empty bins/tails). Provide uncertainty estimates for skewness/kurtosis (e.g., bootstrap CIs) and/or show robustness to trimming the most extreme fraction of cells (e.g., top/bottom 0.1%). Ensure moments correspond to the same variable whose PDF is shown (ρ vs $\log \rho$, etc.).

8. **Turbulence and regime-identification methods are introduced but not yet quantitatively or reproducibly supported: correlation/structure-function implementation details are sparse and results are not clearly reported; spectra are discussed qualitatively without fitted slopes/ranges and with unclear boundary/windowing treatment; k-means clustering lacks feature scaling details and quantitative validation of $k = 4$ (Sec. 2.2.4; Sec. 2.3.3–2.3.4; Sec. 3.2.1–3.2.2; Fig. 6).**

Recommendation: For correlations/structure functions (Sec. 2.2.4), specify sampling strategy (number of pairs, separations, directional sampling), boundary handling, and whether masks break FFT assumptions; then either present the main quantitative outcomes (e.g., correlation lengths, anisotropy measures) or explicitly de-emphasize/remove these methods if inconclusive. For spectra (Sec. 2.3.3; Sec. 3.2.1; Fig. 6), document detrending/windowing/mean subtraction and non-periodic boundary treatment; fit and report approximate slopes over stated k -ranges with caveats about limited inertial range. For clustering (Sec. 2.3.4; Sec. 3.2.2), list features, normalization (e.g., z-score), subsampling, explored k -range, and show/summarize silhouette/elbow metrics; add cluster mass/volume fractions and typical parameter ranges to substantiate physical interpretation.

Minor issues

1. Internal section cross-references are frequently incorrect/inconsistent (e.g., region masks referenced as Sec. 1.3 vs actually Sec. 2.1.2; Q-criterion and streamlines referencing wrong sections; force-balance referenced as “Section 5”) (Sec. 2-3).

Recommendation: Perform a global audit of all intra-paper references after finalizing numbering. Ensure region definitions point to Sec. 2.1.2, streamlines to Sec. 2.4, Q-criterion to Sec. 2.3.2, spectra to Sec. 2.3.3, clustering to Sec. 2.3.4, and force/radiation diagnostics to Sec. 2.5.

2. “Flux quantification” via “streamlines crossing a control surface” is not mathematically defined and risks confusing streamline counts with Eulerian surface flux (related to the headline L1 result) (Sec. 2.4; Sec. 3.3).

Recommendation: Define the mass-transfer estimator explicitly. Prefer direct discrete surface integrals of $\mathbf{J} \cdot \mathbf{n}$ over grid faces/cells for \dot{M} . If a streamline-based estimator is retained, specify the weighting that makes it consistent with a surface integral (seed density tied to area elements and local flux).

3. The Q-criterion/vortex analysis lacks derivative-scheme details and robustness to threshold choice; the reported “ $\sim 2.5\%$ volume with $Q > 0$ ” is method-dependent (Sec. 2.3.2; Sec. 3.2.1).

Recommendation: State the gradient operator (finite-difference order / spectral), any smoothing/filtering, and how spherical geometry terms are treated. Provide a brief sensitivity check varying the Q threshold (or report a Q PDF) to contextualize the 2.5% figure.

4. Figure/caption clarity: axes often lack units and log base; weighting (volume vs mass) and “single snapshot” nature are not always stated; some plots would benefit from explicit markers for donor, companion, and L1 location and a clear definition of the “L1 vicinity” region (Figs. 3–11; Sec. 3).

Recommendation: Standardize figure annotations: units, log base, weighting, and region definitions in captions; add clear geometric markers (donor center, companion direction, L1 point/plane) where relevant; add panel labels (a,b,c,...) and reference them in text.

5. Notation and labeling are inconsistent across text/figures/tables (v_1 vs v_{11} ; \mathbf{J}_r vs \mathbf{J}_r ; J_{mag} vs $|\mathbf{J}|$; spherical vs Cartesian component labels) (Sec. 2.2; Figs. 3-5; Table 1).

Recommendation: Create a concise symbol glossary and enforce a single notation set across the manuscript. Ensure component labels match the coordinate basis used and correct any mixed spherical/Cartesian naming in formulas and captions.

6. Claims of log-normal/power-law behavior in PDFs and spectra are largely qualitative; fitting ranges, parameters, and model-selection metrics are absent (Sec. 3.1; Sec. 3.2.1; Figs. 3–4, 6).

Recommendation: If such claims remain in the narrative, add simple quantitative fits (with stated ranges) and report fitted parameters with uncertainties; otherwise, rephrase to “heavy-tailed” / “approximately power-law-like over a limited range” and avoid implying strong model discrimination from one snapshot.

7. “Benchmark” framing is underspecified: it is unclear which exact diagnostics constitute the benchmark and for what parameter regime it applies (Abstract; Sec. 4.1–4.3).

Recommendation: Explicitly enumerate benchmark outputs (e.g., Table 1 moments, Table 2 force ratios, Fig. 6 spectral slopes over specified k -range, cluster fractions) and state the regime of applicability (donor/binary parameters, radiation closure).

8. Reproducibility support is incomplete: file/tool mentions are not enough to reproduce gradients, $\nabla \cdot \mathbf{P}_r$, masks, spectra, and streamline procedures (Sec. 2).

Recommendation: Provide a minimal reproducibility package (or appendix): parameter values, region mask definitions, pseudocode or scripts for derivative operators in the chosen coordinates (including metric terms), spectra computation choices (windowing/boundaries), and streamline integrator settings.

9. Tone/metadata: informal or nonstandard affiliation/wording (e.g., playful affiliation lines; overly promotional phrasing like “unprecedented”) is not appropriate for many journals (title block; Abstract; Sec. 1; Sec. 4).

Recommendation: Replace informal affiliations with journal-appropriate institutional information and revise promotional language to neutral scientific phrasing.

Very minor issues

1. Typographical/LaTeX artifacts and minor equation formatting errors (e.g., corrupted $|\mathbf{J}|$ definition text; inconsistent code-name quoting; heading artifacts like “# 3.2.2.”) appear in Sec. 2-3 and Tables/Figures.

Recommendation: Proofread and clean all LaTeX/equation rendering; typeset key definitions as numbered display equations; standardize formatting for code/file names and headings.

2. Keyword list formatting and specificity could be improved (Abstract).

Recommendation: Reformat into a standard keyword section and prioritize domain-specific terms (RSGs, binary mass transfer, RLOF, radiation hydrodynamics, convection/turbulence) over very generic keywords.

3. Figure accessibility: some panels have small fonts/low DPI or non-colorblind-safe palettes, and captions can be overly long/redundant.

Recommendation: Export figures as vector/high-DPI, increase font sizes, use colorblind-safe palettes, and streamline captions to essential information plus precise definitions.

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: substantial

The paper is method-heavy and uses a set of core mathematical definitions and operators (coordinate transforms; flux definitions; PDF moments; correlation/structure functions; tensor decompositions and Q-criterion; Mach number and sound speed; force-density terms including $\nabla \cdot P_r$ and $\nabla \cdot F_r$; streamline ODE integration). There are few step-by-step derivations, so the audit focuses on definition consistency, coordinate/basis consistency, and whether the stated operators are sufficiently specified to be verifiable. The most consequential internal issue is the unclear/mixed use of spherical vs Cartesian components in vector magnitudes and streamline tracing; additional key operator formulas in spherical coordinates are not provided, preventing verification of vortex/radiation-force calculations.

Checked items

1. ✓ Spherical-to-Cartesian coordinate transform (Sec. 2.1.1, p.3)

- **Claim:** Cartesian coordinates are computed from spherical (r, θ, ϕ) via $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$.
- **Checks:** algebra, notation consistency
- **Verdict:** PASS; confidence: high; impact: moderate
- **Assumptions/inputs:** r is radial coordinate, θ polar angle, ϕ azimuthal angle as stated, Standard spherical coordinate convention is used
- **Notes:** The stated transform is internally consistent with the declared coordinate meaning and later statements about the orbital plane being x - y ($\theta = \pi/2$).

2. ✓ L1 point and analysis-region definitions (Sec. 2.1.2, p.3)

- **Claim:** Defines RSG surface shell ($650 < r < 684$), L1 vicinity cube centered at $x = 1073.9$ with side 200, and an initial accretion-stream mask based on r , $vel1$, and ρ thresholds.
- **Checks:** definition consistency, sanity/limiting check
- **Verdict:** PASS; confidence: medium; impact: minor
- **Assumptions/inputs:** $rm2 = 2000$ in simulation units, $r11$ factor 0.536974 is taken as given
- **Notes:** Internally consistent placement (L1 between donor at $x = 0$ and companion at $x = 2000$). The numerical factor for $r11$ is not derived here but is used consistently.

3. ✓ Mass flux vector definition (Sec. 2.2.1, p.3)

- **Claim:** Defines instantaneous mass flux vector field \vec{J} as $\vec{J} = \rho \vec{v}$ with components $(\rho \cdot vel1, \rho \cdot vel2, \rho \cdot vel3)$.
- **Checks:** definition consistency, dimensional consistency
- **Verdict:** PASS; confidence: high; impact: moderate
- **Assumptions/inputs:** $vel1$ - $vel3$ are the three velocity components on the grid (stated earlier as spherical components)
- **Notes:** Component-wise multiplication is consistent. Direction of \vec{J} equals direction of \vec{v} for $\rho > 0$.

4. ✗ Mass flux magnitude expression vs component labels (Sec. 2.2.1, p.3)

- **Claim:** Defines $|\vec{J}|$ as $\sqrt{J_x^2 + J_y^2 + J_z^2}$ while constructing components from $(vel1, vel2, vel3)$.
- **Checks:** notation consistency, coordinate/basis consistency
- **Verdict:** FAIL; confidence: high; impact: moderate
- **Assumptions/inputs:** $vel1$ - $vel3$ are spherical components (radial/polar/azimuthal) as stated in multiple places
- **Notes:** The paper mixes Cartesian labels (J_x, J_y, J_z) with a component definition based on $(vel1, vel2, vel3)$ that are described as spherical components. If the intent is simply the Euclidean norm in an orthonormal spherical basis, the correct notation would be $\sqrt{J_r^2 + J_\theta^2 + J_\phi^2}$, or else J must be converted to Cartesian components before using x, y, z subscripts.

5. ✓ Radial mass flux definition (Sec. 2.2.2 and Sec. 3.1.1, pp.3,7)

- **Claim:** Defines radial mass flux $J_r = \rho \cdot vel1$.
- **Checks:** definition consistency
- **Verdict:** PASS; confidence: high; impact: minor
- **Assumptions/inputs:** $vel1$ denotes the radial velocity component in spherical coordinates
- **Notes:** Consistent with the stated meaning of $vel1$ as radial velocity in spherical coordinates and with the later skewness discussion.

6. ✓ Density fluctuation field for correlations (Sec. 2.2.4(1), p.4)

- **Claim:** Defines $\delta = (\rho - \langle \rho \rangle) / \langle \rho \rangle$ for use in two-point correlations.
- **Checks:** definition consistency
- **Verdict:** PASS; confidence: high; impact: minor

- **Assumptions/inputs:** $\langle \rho \rangle$ is the mean over the region/domain used
 - **Notes:** Standard nondimensional fluctuation definition; internally coherent.
7. \triangle **FFT-based two-point correlation procedure** (Sec. 2.2.4(1), p.4)
- **Claim:** Computes correlation by $FFT(\delta)$, takes squared magnitude (power spectrum), then inverse FFT to yield correlation function.
 - **Checks:** derivation logic, assumption completeness
 - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
 - **Assumptions/inputs:** Discrete FFT conventions and domain boundary conditions are suitable for the intended correlation, No masking artifacts or appropriate treatment if masked
 - **Notes:** The Wiener–Khinchin-style relationship requires specific assumptions (notably how boundaries/periodicity are handled and normalization). The paper does not specify these, so the exact meaning/normalization of the resulting “correlation function” cannot be verified from the text.
8. \checkmark **Second-order velocity structure function** (Sec. 2.2.4(2), p.4)
- **Claim:** Defines $S_2(\vec{l}) = \langle |\vec{v}(\vec{x} + \vec{l}) - \vec{v}(\vec{x})|^2 \rangle$.
 - **Checks:** definition consistency
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** A consistent definition of \vec{v} and averaging domain is used
 - **Notes:** Standard definition; no internal contradiction.
9. \triangle **Velocity gradient tensor in spherical coordinates** (Sec. 2.3.1, p.4)
- **Claim:** Computes $\nabla \vec{v}$ (3×3 tensor) from discrete spherical-grid data using finite differences in $x_1 v, x_2 v, x_3 v$.
 - **Checks:** assumption completeness, coordinate/basis consistency
 - **Verdict:** UNCERTAIN; confidence: medium; impact: moderate
 - **Assumptions/inputs:** Correct spherical-coordinate gradient formulas (including geometric terms) are used, Component/basis conventions for the tensor are consistent with later S/Ω decomposition
 - **Notes:** No explicit component expressions are provided. In spherical coordinates, derivatives of vector components generally involve geometric/scale-factor terms; without the formulas, internal correctness cannot be audited.
10. \checkmark **Q-criterion definition** (Sec. 2.3.2, p.4)
- **Claim:** Defines $Q = \frac{1}{2}(\|\Omega\|_F^2 - \|S\|_F^2)$ with Ω antisymmetric and S symmetric parts of $\nabla \vec{v}$.
 - **Checks:** algebra, definition consistency
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** S and Ω are computed from the same well-defined velocity gradient tensor
 - **Notes:** Algebraic definition is consistent given a correctly defined $\nabla \vec{v}$. (Correctness of $\nabla \vec{v}$ itself is separately UNCERTAIN.)
11. \checkmark **Mach number and sound speed** (Sec. 2.3.4(1), p.5)
- **Claim:** Defines $M = |\vec{v}|/c_s$ with $c_s = \sqrt{\gamma P_g/\rho}$, $\gamma = 5/3$.
 - **Checks:** dimensional consistency, definition consistency
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** P_g and ρ are consistent thermodynamic variables in the simulation snapshot
 - **Notes:** Dimensionally consistent; standard ideal-gas/adiabatic sound speed form.
12. \checkmark **Kinetic energy density used in spectra** (Sec. 2.3.3(1), p.4)
- **Claim:** Defines $E_k = \frac{1}{2}\rho|\vec{v}|^2$ for spectral analysis.
 - **Checks:** dimensional consistency, definition consistency
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** E_k is intended as kinetic energy per volume (density of kinetic energy)
 - **Notes:** Consistent as an energy density; appropriate for comparing spatial spectra of energetics.
13. \checkmark **Force-density terms in local force balance** (Sec. 2.5.1 and Sec. 3.4.1, pp.6,11)
- **Claim:** Defines $\text{\$}\vec{F}\{\text{\$}\}_{\text{\$}rm gas\text{\$}} = -\nabla P_g, \text{\$}\vec{F}\{\text{\$}} = -\rho\nabla\Phi, \text{\$}\vec{F}\{\text{\$}\}_{\text{\$}rm inertial\text{\$}} = -\rho(\vec{v}\dot{\cdot}\nabla)\vec{v}, \text{\$}\vec{F}\{\text{\$}} = \nabla\cdot P_r\text{\$}$ and compares magnitudes via ratios.
 - **Checks:** dimensional consistency, symbol consistency
 - **Verdict:** PASS; confidence: medium; impact: moderate
 - **Assumptions/inputs:** All forces are treated as force per unit volume fields, Sign is irrelevant for magnitude ratios
 - **Notes:** As written, these are dimensionally comparable as force densities. However, the paper does not specify whether additional radiation-momentum terms (if any in the employed formulation) are neglected; within the paper’s own stated comparison framework, the definitions are consistent.
14. \triangle **Radiation force computation as divergence of P_r in spherical coordinates** (Sec. 2.5.1(4), p.6)

- **Claim:** Computes $\vec{F}_{\text{rad}} = \nabla \cdot P_r$ using the full divergence formula for a symmetric tensor in spherical coordinates.
 - **Checks:** assumption completeness, coordinate/basis consistency
 - **Verdict:** UNCERTAIN; confidence: low; impact: critical
 - **Assumptions/inputs:** P_r components ($P_{r11}, P_{r12}, \dots, P_{r33}$) correspond to the same coordinate basis used in the divergence operator, Correct spherical tensor-divergence expressions are used
 - **Notes:** No explicit divergence formula is provided, and the coordinate basis of $P_{r,ij}$ is not stated (spherical vs Cartesian). Since force-ratio maps and main conclusions hinge on $|\vec{F}_{\text{rad}}|$, the absence of these definitions prevents an internal mathematical audit.
15. ✓ **Radiative heating/cooling definition and sign convention** (Sec. 2.5.4, p.6 and Sec. 3.4.2, p.12)
- **Claim:** Defines $Q_{\text{rad}} = \nabla \cdot \vec{F}_{\text{rad}}$ and $Q_{\text{rad}} > 0$ as cooling, $Q_{\text{rad}} < 0$ as heating (energy absorbed by gas) and Q_{rad}
 - **Checks:** definition consistency
 - **Verdict:** PASS; confidence: medium; impact: minor
 - **Assumptions/inputs:** The paper's sign convention for energy exchange is as stated
 - **Notes:** The interpretation is consistent with the paper's stated convention; the governing energy equation is not shown, but no internal contradiction appears.
16. ✓ **Temperature proxy** (Sec. 2.5.4(2), p.6)
- **Claim:** Approximates temperature as $T \propto P_g/\rho$ for correlation analysis.
 - **Checks:** dimensional/physical proxy sanity
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** Only proportionality is used (constants omitted)
 - **Notes:** As a proxy up to constants, P_g/ρ is consistent with an ideal-gas-like scaling; proportionality (not equality) is correctly stated.
17. ✗ **Streamline tracing definition for \vec{J} and coordinate system** (Sec. 2.4.2, p.5; Sec. 3.3, p.10; Figs. 7-8)
- **Claim:** Traces streamlines of $\vec{J} = \rho \vec{v}$ from seed points on $r = 684$ surface and assesses whether they cross the plane $x = 1073.9$.
 - **Checks:** coordinate/basis consistency, definition completeness
 - **Verdict:** FAIL; confidence: high; impact: critical
 - **Assumptions/inputs:** The streamline ODE uses a vector field expressed in the same coordinates as the integrated position, Interpolation is performed consistently on the underlying grid
 - **Notes:** The paper describes \vec{v} components as spherical (vel1,vel2,vel3) and constructs \vec{J} from them, but does not state (or show) conversion to a Cartesian vector field before integrating trajectories in Cartesian space and testing crossing of $x = \text{const}$. Without explicit basis conversion or an explicit spherical-coordinate integration scheme, the streamline geometry (and thus the key claim '0/10,000 cross L1 plane') is not mathematically supported by the provided definitions.
18. △ **Surface flux / mass-transfer-rate estimation via streamlines** (Sec. 2.4.3(3), p.5)
- **Claim:** Estimates instantaneous mass transfer rate through a control surface by summing the mass flux associated with streamlines crossing the surface.
 - **Checks:** derivation logic, assumption completeness
 - **Verdict:** UNCERTAIN; confidence: medium; impact: moderate
 - **Assumptions/inputs:** A mapping exists from discrete streamlines to a surface integral of $\vec{J} \cdot \hat{n}$
 - **Notes:** The method to convert streamline crossings into a quantitative flux is not specified (e.g., required weighting by seed-area elements and local $\vec{J} \cdot \hat{n}$). As written, it is not auditable whether the estimator corresponds to a well-defined discretization of a surface integral.

Limitations

- The audit is based only on the provided PDF text content; several computations are described procedurally without explicit equations (notably spherical-coordinate derivatives and tensor divergences), preventing full symbolic verification.
- No governing PDEs (momentum/energy equations) are written in the paper excerpt; checks involving sign conventions for radiation coupling and inertial terms are therefore limited to internal consistency of the paper's stated definitions rather than consistency with an explicit equation set.
- Masking/subdomain operations for FFT-based correlations are described at a high level; without exact discrete definitions and boundary treatments, only partial analytic validation is possible.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

All 21 automated internal-consistency checks passed (arithmetic, rounding, sign consistency, basic range validity, and limited text–table alignment). Several key quantitative claims (e.g., region statistics, vortex-core volume fraction, streamline outcomes, force ratios, and correlations) remain unverified because they depend on underlying simulation fields and region masks not present in the PDF text.

Checked items

1. ✓ **C1** (Page 3, Sec. 2.1.2 (RSG Stellar Surface definition))
 - **Claim:** Surface shell defined as $x1v$ between 650 and 684 simulation units; stated to correspond to approximately 5% of the nominal RSG radius $R_{\text{RSG}} \approx 667$.
 - **Checks:** fraction_of_radius
 - **Verdict:** PASS
 - **Notes:** Computed $(684 - 650)/667 = 0.0509745$, consistent with the stated $\sim 5\%$ within the provided absolute tolerance for an approximate claim.
2. ✓ **C2** (Page 3, Sec. 2.1.2 (RSG Stellar Surface definition))
 - **Claim:** Nominal RSG radius stated as $R_{\text{RSG}} \approx rm2/3 \approx 667$ with $rm2 = 2000$ simulation units.
 - **Checks:** algebraic_consistency
 - **Verdict:** PASS
 - **Notes:** $rm2/3 = 666.666\dots$, which rounds to 667.
3. ✓ **C3** (Page 3, Sec. 2.1.2 (L1 point location))
 - **Claim:** L1 point located at $r11 = 0.536974 \times rm2 = 1073.9$ simulation units, with $rm2 = 2000$.
 - **Checks:** multiplication_consistency
 - **Verdict:** PASS
 - **Notes:** $0.536974 \times 2000 = 1073.948$, consistent with 1073.9 at 1 decimal place.
4. ✓ **C4** (Page 3, Sec. 2.1.2 (L1 vicinity cube))
 - **Claim:** L1 vicinity defined as a cubic volume of side length 200 simulation units centered on $x = 1073.9$.
 - **Checks:** derived_bounds
 - **Verdict:** PASS
 - **Notes:** Derived x -extent computed as [973.9, 1173.9].
5. ✓ **C5** (Page 5, Sec. 2.3.4 (Mach number definition))
 - **Claim:** Sound speed defined as $c_s = \sqrt{\gamma P_g / \rho}$ with $\gamma = 5/3$.
 - **Checks:** constant_value_check
 - **Verdict:** PASS
 - **Notes:** Identity/representation consistency check: $5/3$ matches 1.666666... to within floating rounding.
6. ✓ **C6** (Page 8, Table 1 (RSG Surface, ρ mean and std from variance))
 - **Claim:** For RSG Surface density: mean = 8.49×10^{-4} and variance = 5.57×10^{-6} .
 - **Checks:** variance_to_std
 - **Verdict:** PASS
 - **Notes:** Derived quantities computed: std = 0.0023600847; cv = 2.7798407 (no stated target to compare against).
7. ✓ **C7** (Page 8, Table 1 (RSG Surface, vel1 variance to std))
 - **Claim:** For RSG Surface vel1: variance = 0.070 and mean = -0.115 .
 - **Checks:** variance_to_std
 - **Verdict:** PASS
 - **Notes:** Derived quantities computed: std = 0.2645751311; cv (std/|mean|) = 2.3006533140 (no stated target to compare against).
8. ✓ **C8** (Page 8, Table 1 (L1 Vicinity, J_r mean sign vs stated skewness sign narrative))
 - **Claim:** L1 vicinity J_r mean is negative (-3.93×10^{-8}) and skewness is negative (-3.17); narrative says net flow back towards donor.
 - **Checks:** sign_consistency
 - **Verdict:** PASS
 - **Notes:** Both mean and skewness are negative as stated.
9. ✓ **C9** (Page 8, Table 1 (Accretion Stream, J_r mean sign vs skewness sign))
 - **Claim:** Accretion Stream J_r mean is positive (4.01×10^{-3}) and skewness positive (5.85); narrative says net outflow.
 - **Checks:** sign_consistency
 - **Verdict:** PASS
 - **Notes:** Both mean and skewness are positive as stated.

10. ✓ **C10** (Page 8 (text below Table 1) and Page 9 (Sec. 3.2.1))
 - **Claim:** Only 2.50% of the simulation volume consists of vortex cores (regions where $Q > 0$).
 - **Checks:** percentage_to_fraction
 - **Verdict:** PASS
 - **Notes:** 2.50% converts exactly to 0.025.
11. ✓ **C11** (Page 10, Sec. 3.3 and Sec. 3.3.1)
 - **Claim:** Streamline tracing used 10,000 seed points; zero out of 10,000 streamlines crossed the L1 control surface.
 - **Checks:** count_to_rate
 - **Verdict:** PASS
 - **Notes:** Computed crossing fraction $0/10000 = 0$ (0%).
12. ✓ **C12** (Page 10, Sec. 3.3 (seed radius) vs Page 3, Sec. 2.1.2 (surface shell outer bound))
 - **Claim:** Seeds placed on RSG surface at $r = 684$ simulation units; surface shell outer boundary is 684.
 - **Checks:** repeated_constant_match
 - **Verdict:** PASS
 - **Notes:** Exact constant match ($684 = 684$).
13. ✓ **C13** (Page 10, Sec. 3.3.2)
 - **Claim:** Mean tortuosity reported as 1.000009; stated most streamlines close to 1.0.
 - **Checks:** difference_from_unity
 - **Verdict:** PASS
 - **Notes:** Delta from unity computed as 9×10^{-6} , consistent with the narrative.
14. ✓ **C14** (Page 12, Sec. 3.4.1 and Page 13, Table 2 (RSG Surface mean ratio))
 - **Claim:** Text says mean $\frac{|\vec{F}|}{|\vec{F}|_{onRSG\ Surface}} \approx 0.005$; Table 2 lists 0.0048\$.
 - **Checks:** text_table_rounding_consistency
 - **Verdict:** PASS
 - **Notes:** Table value 0.0048 is close to the text approximation 0.005 (4% relative difference).
15. ✓ **C15** (Page 12, Sec. 3.4.1 and Page 13, Table 2 (L1 Vicinity mean ratio))
 - **Claim:** Text says in L1 vicinity radiation force is on average 3.5 times stronger than gas pressure force; Table 2 mean is 3.51.
 - **Checks:** text_table_rounding_consistency
 - **Verdict:** PASS
 - **Notes:** 3.51 is consistent with '3.5 times' rounding.
16. ✓ **C16** (Page 12, Sec. 3.4.1 and Page 13, Table 2 (L1 Vicinity max ratio))
 - **Claim:** Text says in L1 vicinity radiation force is up to 210 times stronger in localized regions; Table 2 max is 210.5.
 - **Checks:** text_table_rounding_consistency
 - **Verdict:** PASS
 - **Notes:** 210.5 is consistent with the approximate text value 210.
17. ✓ **C17** (Page 13, Table 2 (Accretion Stream mean ratio formatting))
 - **Claim:** Accretion Stream mean $\frac{|\vec{F}|}{|\vec{F}|_{reported}} = 5.00 \times 10^6$.
 - **Checks:** scientific_notation_parse
 - **Verdict:** PASS
 - **Notes:** Scientific notation parses to 5,000,000.
18. ✓ **C18** (Page 13, Table 2 (Accretion Stream max ratio formatting))
 - **Claim:** Accretion Stream max $\frac{|\vec{F}|}{|\vec{F}|_{.}} = 1.27 \times 10^{10}$
 - **Checks:** scientific_notation_parse
 - **Verdict:** PASS
 - **Notes:** Scientific notation parses to 12,700,000,000.
19. ✓ **C19** (Page 13, Table 2 (Mean vs max ordering per region))
 - **Claim:** In Table 2, max ratios should be \geq mean ratios for each region and each ratio type.
 - **Checks:** inequality_consistency
 - **Verdict:** PASS
 - **Notes:** All max \geq mean inequalities evaluated true for the provided Table 2 values.
20. ✓ **C20** (Page 12, Sec. 3.4.2 and Page 13, Table 2)

- **Claim:** Text says in Accretion Stream region mean $\frac{|\vec{F}_{\text{rad}}|}{|\vec{F}|} \approx 5 \times 10^{-6}$ (order of magnitude); Table 2 gives 5.00×10^{-6} .
- **Checks:** text_table_exact_match
- **Verdict:** PASS
- **Notes:** After notation normalization, the text and table values match exactly.

21. ✓ **C21** (Page 12, Sec. 3.4.2 (Pearson correlations))

- **Claim:** Correlation between $|\vec{F}_{\text{rad}}|$ and $|J|$ reported as $r = 0.43$ on RSG surface and $r = 0.30$ in accretion stream.
- **Checks:** range_check
- **Verdict:** PASS
- **Notes:** Both reported Pearson r values are within the valid range $[-1, 1]$.

Limitations

- Only the PDF text was available; no access to the referenced HDF5 snapshot, grids, or derived fields, so most numerical claims tied to computed statistics/fields cannot be recomputed.
- Figures are treated as non-numeric (no pixel/plot-value extraction); checks avoid reading values from plots.
- Some quantities mix 'simulation units' with 'solar radii' in narrative; checks are limited to internal arithmetic/rounding consistency, not physical unit validation.