Skepthical review: *Quantifying Spin-Dependent Yarkovsky Drift: Empirical Evidence from Asteroid Family V-Shapes*

Summary

The paper extends classical asteroid-family V-shape analysis, which relates semimajor-axis dispersion $|\Delta a|$ to inverse diameter $1/D$ as a Yarkovsky signature, to explicitly include spin period. The authors construct a merged dataset of $\approx 15,700$ asteroids from multiple catalogs, with family membership, semimajor axis, diameter, spin period, and family age (Sec. 2.1–2.2; Sec. 3.1). For families with ≥ 50 members and complete data (33 in the final analysis), they estimate a central family semimajor axis a_{center} using kernel density estimation, define $|\Delta a| = |a - a_{\text{center}}|$, and fit the upper V-shape boundaries in three spaces—$1/D$, $1/P$, and $1/(D \cdot P)$—using a binned-maxima, weighted linear regression constrained through the origin (Sec. 2.3.1–2.3.3). This yields three empirical drift coefficients per family: k_D, k_P, and k_{PD} (Sec. 3.2).

The results show visually clear V-shapes in many families across all three parameter spaces (Sec. 3.2.1; Figs. 3–21). The fitted k-values are summarized in Table 1 (Sec. 3.2.2). Correlating k_D, k_P, and k_{PD} with literature family ages (Sec. 2.4; Sec. 3.3), the authors find statistically significant positive correlations, with Pearson coefficients $r \approx 0.63$ for k_D vs. age, $r \approx 0.49$ for k_P vs. age, and $r \approx 0.62$ for k_{PD} vs. age and p-values $\lesssim 10^{-3}$ (Sec. 3.3.1–3.3.3). They interpret these trends as empirical evidence that spin period, in addition to size, modulates long-term Yarkovsky-driven semimajor-axis dispersion, and suggest that the combined size–spin coefficient k_{PD} may act as a unified chronometer. Limitations—including simplified linear modeling, missing obliquity/YORP physics, incomplete and potentially biased spin data, and neglected measurement/age uncertainties—are discussed qualitatively (Sec. 3.4; Conclusion).

Overall, the study is clearly motivated and provides a conceptually simple, reproducible framework for probing spin-dependent Yarkovsky evolution at the family level. The approach is applied uniformly to a reasonably large sample and yields correlations broadly consistent with expectations from Yarkovsky theory. However, internal inconsistencies in reported dataset sizes, incomplete specification and validation of the boundary-fitting and regression procedures, lack of uncertainty propagation, limited treatment of selection effects, and somewhat strong claims about "physically grounded chronometers" for spin-related coefficients currently limit the robustness and interpretability of the conclusions. Addressing these issues would substantially strengthen the work's credibility and impact.

Strengths

- Clearly motivated extension of classical $1/D$ V-shape analysis to include spin period and a combined size–spin proxy, directly targeting empirical spin dependence of Yarkovsky-driven dispersion (Sec. 1; Sec. 2.3.1; Sec. 3.2).

- Conceptually simple, uniform, and reproducible boundary-fitting framework (binned maxima plus weighted linear regression constrained through the origin) applied consistently across three proxy spaces, facilitating systematic comparison between k_D, k_P, and k_{PD} (Sec. 2.3.3; Sec. 3.2).
- Use of a relatively large, systematically constructed dataset ($\approx 15,700$ asteroids in 62 families, with 33 families meeting the ≥ 50-member complete-data criterion), enabling a comparative analysis of Yarkovsky signatures across many families (Sec. 2.1–2.2; Sec. 3.1; Table 1).
- Presentation of both qualitative visual evidence (V-shapes in numerous families) and quantitative correlations of k_D, k_P, and k_{PD} with family age, with statistically significant r-values that align with expectations of cumulative Yarkovsky drift (Sec. 3.2.1–3.2.2; Sec. 3.3.1–3.3.3).
- Thoughtful qualitative discussion of key limitations—data incompleteness, selection effects, simplified linear physics, neglected obliquity and YORP, and thermophysical diversity—which shows awareness of caveats and points toward future improvements (Sec. 3.4; Conclusion).
- The figures systematically and clearly compare size, spin, and combined-proxy V-shapes across asteroid families, using consistent visual encoding (gray points, red boundaries), clear panel layouts, and overlays that effectively communicate both qualitative and quantitative aspects of Yarkovsky-driven dispersion.
- Core mathematical definitions are simple and largely self-consistent: the orbital drift proxy is consistently defined as $\Delta a = |a - a_{\text{center}}|$ and modeled linearly against inverse physical proxies ($1/D$, $1/P$, $1/(D \cdot P)$).
- Dimensional/unit consistency of the fitted slope parameters (k_D, k_P, k_{PD}) matches the stated regression model $\Delta a_{\max} = k \cdot (1/Y)$: k has units of AU·Y, consistent with Table 1 headings and later unit discussions.
- The zero-intercept constraint is analytically consistent with the chosen dependent variable Δa (not a), since $\Delta a \to 0$ as $1/Y \to 0$ under the paper's linear envelope model.

Major issues

1. **The description of dataset construction is internally inconsistent between Methods and Results, leading to confusion about the actual number of asteroids and families analyzed (Sec. 2.2 vs. Sec. 3.1, plus Abstract and Conclusion). Sec. 2.2 mentions 38,451 asteroids in 118 families, reduced to 42 families, whereas Sec. 3.1 states 15,749 asteroids in 62 families, reduced to 33 families. These discrepancies propagate into Table 1 and the k–age correlation analysis in Sec. 3.3 and undermine reproducibility and interpretation of sample size, selection effects, and statistical power.**

Recommendation: Unify and document the full data curation pipeline. In Sec. 2.1–2.2 and Sec. 3.1, provide a single consistent sequence: (i) initial counts per source catalog (asteroids and families); (ii) counts after each filtering step (e.g., removal of missing D , P, a, or age; quality cuts; family merges); (iii) final number of asteroids and families used for V-shape fitting; and (iv) subset used for k–age correlations. Correct any erroneous numbers and ensure that Abstract, Table 1, Sec. 3.1, Sec. 3.3, and the Conclusion all quote the same finalized counts. Consider adding a flow diagram or summary table of these numbers.

2. **The boundary model $\Delta a_{\text{max}} = k \cdot (1/Y)$ with a forced zero intercept, implemented via a binned-maxima, weighted linear regression, is central to defining k_D, k_P, and k_{PD} but is insufficiently specified and not quantitatively validated (Sec. 2.3.1–2.3.3; Sec. 3.2.2). The diurnal Yarkovsky effect has non-linear dependence on spin period and strong obliquity dependence, so a strictly linear, zero-intercept relation is a simplification that may bias k. Moreover, key implementation details—bin number/edges, minimum bin population, exact weighting scheme, regression routine, and treatment of Δa_{max} uncertainties—are only described qualitatively, and no sensitivity tests to these choices or to the zero-intercept constraint are presented.**

   Recommendation: In Sec. 2.3.3, fully specify the boundary-fitting algorithm: (i) define how the number of bins is chosen (fixed N or a function of sample size) and whether bins are equal-width in $1/Y$; (ii) give the formula for Δa_{max} per bin and the weights used in the regression (e.g., $w_j = N_j$); and (iii) state the regression method and software. Then, in Sec. 3.2.2 (or a new subsection), present robustness tests on several well-populated families: vary bin counts (e.g., 10, 15, 20, 30), minimum bin population, and whether an intercept is free or constrained to zero, and quantify how k_Y changes and how goodness-of-fit metrics (e.g., R^2, residuals) behave. Discuss in Sec. 3.3 how potential systematic biases from these modeling choices could influence the inferred age correlations.

3. **Measurement uncertainties in diameters, spin periods, and especially family ages, as well as heterogeneity in age determinations, are acknowledged only qualitatively and are not propagated through to k_Y estimates or to the k_Y–age regressions (Sec. 2.1–2.4; Sec. 3.3; Sec. 3.4). All correlations appear to be treated with simple linear regression and Pearson r without accounting for errors-in-variables, confidence intervals on k_Y, or uncertainties on ages. This can lead to underestimated uncertainties and potentially overconfident assessment of correlation strengths, particularly where scatter is large (e.g., k_P).**

   Recommendation: Augment Sec. 2.4 and Sec. 3.3 with explicit uncertainty handling: (i) summarize typical or catalog-provided uncertainties for D, P, and family ages, citing sources; (ii) where feasible, propagate D and P errors into Δa and the $1/Y$ proxies, or justify approximations; and (iii) perform bootstrap or Monte Carlo simulations in which D, P, and ages are perturbed within their uncertainties to obtain distributions for k_D, k_P, k_{PD} and for the regression slopes and Pearson r-values. Report confidence intervals (e.g., 68% or 95%) on k_Y and on the k_Y–age relations in Table 1 or supplementary material, and temper claims in Sec. 3.3 and the Conclusion where significance is sensitive to uncertainties.

4. **Selection effects and biases in the availability of spin periods, completeness of family membership, and adopted family ages are only briefly mentioned and not quantified (Sec. 2.2; Sec. 3.1; Sec. 3.4). The requirement of complete spin period data and a ≥ 50-member threshold likely biases the sample toward larger, brighter, and better-studied objects and families, and may correlate with age, heliocentric distance, or dynamical environment. Without a quantitative bias assessment, it is unclear how representative the 33 analyzed families are and how selection might influence the observed k_P and k_{PD} trends and even the k_D–age correlation.**

   Recommendation: In Sec. 2.2 and Sec. 3.1, add a quantitative analysis of selection effects: (i) compare distributions of age, heliocentric distance, and typical size between included and excluded families; (ii) show histograms of diameter and spin period for the analyzed sample versus the full set of family members with known D or a; and (iii) discuss how these differences might bias k_P and k_{PD} (e.g., preferential sampling of families with richer spin coverage or particular size ranges). If possible, recompute the k_D–age relation using a broader set of families that only require D (not P) to test for selection-induced distortions. Emphasize in Sec. 3.4 and the Conclusion that current correlations apply to this biased subset and may not generalize to all families.

5. **The physical interpretation of k_P and k_{PD}, and their relation to underlying Yarkovsky and YORP physics, remains somewhat underdeveloped and partially inconsistent with the complexities acknowledged later (Sec. 1; Sec. 3.2.2; Sec. 3.3.2–3.3.3; Sec. 3.4; Conclusion). The linear dependence on $1/P$ is treated as a first-order approximation, but potential non-linearities (e.g., saturation or regime changes for very fast/slow rotators), obliquity distributions, and YORP-driven spin evolution are not examined empirically. As a result, statements that k_P and k_{PD} provide "physically grounded chronometers" risk over-interpretation, given that these coefficients may be better viewed as empirical descriptors integrating multiple processes.**

   Recommendation: Refine the interpretation in Sec. 3.2.2, Sec. 3.3.2–3.3.3, Sec. 3.4, and the Conclusion by: (i) clearly distinguishing k_D, which more directly reflects size-dependent Yarkovsky drift, from k_P and k_{PD}, which are empirical effective parameters combining Yarkovsky, YORP, obliquity distributions, and thermophysical diversity; (ii) exploring non-linearity for a few well-sampled families (e.g., by inspecting $\Delta a_{\max}$

vs. $1/P$ in log–log space or allowing simple curvature/broken-line fits) and commenting on any deviations from linearity; and (iii) softening or qualifying claims that k_P and k_{PD} are new "physically grounded chronometers," presenting them instead as promising empirical proxies whose physical calibration requires further modeling and, where possible, spin-axis information. Include at least one order-of-magnitude comparison between k_D-derived drift rates and canonical Yarkovsky da/dt values to anchor the discussion.

6. **There is no explicit external benchmarking of the derived k_D values or implied drift rates against existing V-shape age estimates or theoretical Yarkovsky calibrations, limiting the ability to assess external validity (Sec. 1; Sec. 3.2.2–3.3.1; Conclusion). While internal k_D–age correlations are presented as a validation of the method, the paper does not show whether k_D for well-studied families (e.g., Vesta-like, Eos, Karin, Veritas) are of the expected magnitude, nor whether inverting k_D reproduces literature ages within uncertainties.**

   Recommendation: In Sec. 3.2.2 and/or Sec. 3.3.1, include an external consistency check: (i) for several benchmark families with published V-shape or dynamical ages, compare the k_D-derived characteristic drift (e.g., via $da/dt \approx k_D/T_{\text{family}}$) to standard Yarkovsky model predictions; (ii) where feasible, invert k_D to estimate ages and compare to literature values; and (iii) briefly discuss how k_P and k_{PD} might be calibrated using theoretical spin-dependent Yarkovsky/YORP models. Use these comparisons to demonstrate that the empirical coefficients are physically reasonable and to clarify any systematic offsets. This will better situate the work within the broader literature and strengthen confidence in the method.

7. **Across Figures 1–21, there are frequent omissions of explicit axis units, normalization details, and key parameter annotations (such as drift coefficients k, their uncertainties, and family center a_c/a_0), as well as a lack of confidence intervals or uncertainty bands on fitted boundaries. These omissions hinder quantitative interpretation, reproducibility, and cross-figure comparison. Additionally, sample sizes, selection criteria, and interloper handling are often not indicated, which can bias the envelope fits and limit the credibility of the results.**

   Recommendation: For all figures, explicitly label axes with symbols and units (e.g., a [AU], $1/D$ [1/km], $1/P$ [1/hr], $1/(D \cdot P)$ [1/(km·hr)]), annotate fitted drift coefficients (k) with uncertainties directly on the panels or in the caption, and mark the family center (a_c/a_0) with a labeled line and value. Add confidence bands (e.g., bootstrap or Monte Carlo) around fitted boundaries, and report sample sizes, selection criteria, and interloper/outlier handling in captions or legends. Ensure all figures use consistent terminology, notation, and visual styling.

8. **Many figures (especially multi-panel layouts) suffer from small physical size, low font and line weights, and overplotting of dense gray points, which together reduce readability, obscure density structure, and make key annotations difficult to discern at print or screen scale. Legends, panel labels, and in-figure explanations are often minimal or absent, requiring readers to rely on captions for symbol/line meanings.**

   Recommendation: Increase figure and panel sizes, font and line weights, and use partial transparency or density/hexbin overlays to reduce overplotting. Add clear panel labels (a, b, c), concise legends or in-panel annotations for symbol/line meanings, and harmonize axis ranges and tick formatting across panels for comparability. Export figures as vector graphics (PDF/SVG) or high-resolution raster to ensure clarity in publication.

9. **Internal inconsistency in dataset and selection counts between Methods and Results: Sec. 2.2 (p.3) states 38,451 asteroids across 118 families with 42 families selected (≥ 50 members), whereas Sec. 3.1 (p.5) and the Abstract state 15,749 asteroids across 62 families with 33 families selected ($\geq$ 50 members). These figures are not reconciled, making it unclear which dataset underlies the mathematical fitting and the reported coefficients/correlations.**

   Recommendation: Add a clear, step-by-step accounting table (or paragraph) showing how the dataset changes across preprocessing steps (e.g., after requiring ages, after requiring spin periods, after family-age join, after filtering), and ensure the counts in Sec. 2.2, Sec. 3.1, and the Abstract refer to the same stage or are explicitly labeled as different stages.

10. **Unresolved internal inconsistency in reported dataset sizes and selected-family counts: some sections report 15,749 asteroids from 62 families with selection to 33 families, while Methods (Sec 2.2) reports 38,451 asteroids among 118 families and selection to 42 families.**

    Recommendation: Reconcile all dataset totals and selection outcomes across Abstract/Methods/Results/Conclusions; explicitly define which dataset each number refers to (source, filters, and final regression/analysis sample) and ensure consistent reporting.

Minor issues

1. The computation of the central semimajor axis a_{center} using kernel density estimation (KDE) is only briefly described and lacks key implementation details and robustness tests (Sec. 2.3.2; Sec. 3.1). The choice of kernel, bandwidth selection method, and

edge handling are not specified, and there is limited quantitative comparison of KDE-based a_{center} to simpler measures (mean/median), especially for families with asymmetric or multi-peaked semimajor-axis distributions.

Recommendation: In Sec. 2.3.2, specify the KDE implementation (e.g., Gaussian kernel, bandwidth determined via Silverman's rule, Scott's rule, or cross-validation; software/library used) and any edge-handling choices. In Sec. 3.1 or Sec. 3.2.2, present a brief quantitative comparison for several representative families, showing differences between KDE peak, mean, and median a and the resulting impact on $|\Delta a|$ and k_Y (e.g., as a fraction of the V-shape width). If bandwidths were tuned manually, describe the criteria; otherwise, state the automatic rule used.

2. Data sources and catalog versions for semimajor axes, diameters, spin periods, family memberships, and family ages are described only generically via CSV filenames, without precise citations or construction details (Sec. 2.1). This hinders reproducibility and makes it difficult to assess the provenance and typical uncertainties of each parameter.

   Recommendation: In Sec. 2.1, explicitly identify the underlying catalogs and references for each quantity (e.g., MPC for orbital elements, WISE/NEOWISE for diameters, LCDB or other lightcurve databases for spin periods, specific family lists and age compilations), including version numbers or retrieval dates. If the CSV files merge multiple sources, describe the merging and conflict-resolution procedure and provide a URL or repository. Consider adding an appendix table summarizing each data source, typical uncertainties, and any selection or quality filters applied.

3. The family selection criterion of ≥ 50 members with complete data, while reasonable, is not accompanied by a characterization of which families are excluded and how this may bias the ensemble (Sec. 2.2; Sec. 3.1). Relatedly, claims that V-shapes appear for "the majority" of the 33 families in all three parameter spaces are not backed by a quantitative criterion defining what constitutes a significant V-shape (Sec. 3.2.1).

   Recommendation: Extend Sec. 2.2 and Sec. 3.1–3.2.1 by: (i) summarizing the properties of excluded families (number, typical ages, sizes, heliocentric distances) and commenting on potential biases introduced by the ≥ 50-member and completeness cuts; and (ii) defining a simple quantitative diagnostic for V-shape presence (e.g., minimum R^2 of the boundary fit, significantly positive slope at $> X\sigma$, or an envelope asymmetry metric) and reporting how many families meet this criterion in each proxy space ($1/D$, $1/P$, $1/(D \cdot P)$). Adjust qualitative statements about "majority" accordingly.

4. Units and scaling conventions for the inverse proxies and drift coefficients are not consistently and explicitly defined at first introduction, which may confuse readers unfamiliar with the setup (Sec. 2.3.1; Sec. 3.2.2; Table 1). For example, $1/D$, $1/P$, and $1/(D \cdot P)$ are introduced without explicit units, while Table 1 later lists units such as AU·km and AU·hr for k_Y.

Recommendation: In Sec. 2.3.1, explicitly define the units of each proxy ($1/D$ in $1/\mathrm{km}$, $1/P$ in $1/\mathrm{hr}$, $1/(D \cdot P)$ in $1/(\mathrm{km} \cdot \mathrm{hr})$) and, in Sec. 2.3.3 or Sec. 3.2.2, state the resulting units for k_D, k_P, and k_{PD}. Provide a compact formula for $\Delta a_{\max}$ per bin and clarify that $\Delta a_{\max}$ versus the bin-center proxy is regressed. Consider adding a short numerical example in Sec. 3.2.2 illustrating how a given k_D translates into an approximate drift for a 1 km asteroid over a given age to help ground the physical interpretation.

5. The description of the k_Y–age correlation analysis omits several statistical details, including the exact regression model, the treatment of intercepts, residual diagnostics, and any consideration of multiple testing across the three coefficients (Sec. 2.4; Sec. 3.3.1–3.3.3). The use of robust regression (RANSAC) is mentioned only qualitatively without numerical comparison to standard fits.

   Recommendation: In Sec. 2.4, clearly state the regression formulation (e.g., ordinary least squares of k_Y on Family_Age with an intercept term, unweighted) and any software used. In Sec. 3.3.1–3.3.3, briefly report whether residuals were inspected for nonlinearity or heteroscedasticity and whether simple multiple-comparison corrections would affect significance. Expand Sec. 3.3.3 with a concise quantitative summary (table or text) comparing slopes, intercepts, Pearson r, and number of inlier families for standard versus RANSAC fits for each k_Y, and note how outliers affect conclusions.

6. The interpretation and wording of some physical statements and age descriptors can be confusing or slightly inconsistent with numerical values, and the link between empirical coefficients and conventional Yarkovsky drift rates is not explicitly demonstrated (Sec. 1; Sec. 3.2.2; Sec. 3.3.2; Sec. 4). For example, some families described as “very young” have Gyr-scale ages in Table 1, and k_D is sometimes described as if it were an instantaneous drift rate rather than an integrated measure.

   Recommendation: Clarify in Sec. 3.2.2 how k_D relates to an average da/dt over the family age (e.g., $da/dt \approx k_D/T_{\text{family}}$ for a 1 km body) and provide at least one worked example comparing to canonical Yarkovsky rates. Harmonize qualitative age descriptors with the numerical ages given in Table 1 (e.g., reserve “very young” for $\lesssim 0.1$ Gyr families or adjust the wording). In the Introduction and Conclusion, ensure that statements about what is “directly measured” focus on empirical correlations rather than detailed physical decomposition, especially for spin-related coefficients.

7. The limitations section provides useful caveats but is somewhat diffuse and not always clearly tied back to specific results, and the Conclusion partially repeats detailed numbers and qualitative statements from Sec. 3.3 without much additional synthesis (Sec. 3.4; Sec. 4).

Recommendation: Restructure Sec. 3.4 into clearer subsections or bullet-style paragraphs (e.g., data limitations, modeling simplifications, missing spin-axis information, selection effects), each explicitly linked to affected results in Sec. 3.2–3.3 and noting how they might bias or weaken specific conclusions. In Sec. 4, reduce repetition of detailed r and p-values already given in Sec. 3.3, and instead emphasize the main robust findings, the most uncertain but intriguing indications, and priority directions for future work (e.g., incorporating obliquities, modeling YORP, extending to more families as spin data improve).

8. Minor inconsistencies exist between textual descriptions and figure captions regarding axis scaling (linear vs. logarithmic inverse proxies) and some captions are highly repetitive across figures (Sec. 3.2.1; Figs. 3–21).

   Recommendation: Verify whether any figures actually use logarithmic scaling of the inverse proxies. If all fits are done in linear $1/Y$, ensure captions and text (e.g., Sec. 3.2.1) consistently describe linear axes; if some figures use log scaling, explicitly state this in Sec. 2.3.1–2.3.3 and clarify that the fitting is performed in linear or log space as appropriate. Streamline later figure captions (Figs. 4–21) by avoiding repeated boilerplate and highlighting what is distinctive about each family (e.g., especially strong/weak spin dependence or unusual k-values).

9. There is inconsistent terminology, notation, and styling across figures and captions (e.g., 'semimajor' vs. 'semi-major', use of dot vs. asterisk in $1/(D \cdot P)$, variable italicization, and legend/line-style descriptions), as well as inconsistent or missing panel labels and axis tick formatting.

   Recommendation: Standardize terminology (e.g., 'semimajor'), notation (use a centered dot for products, consistent variable formatting), legend/line-style descriptions, and panel labeling across all figures and captions. Harmonize axis tick formatting and significant figures to improve polish and reduce confusion.

10. Legends, sample size (N), and data provenance (sources for D and P) are often missing or only in captions, and there is limited indication of outlier/interloper handling or resonance effects. Some figures lack explicit boundary equations or fitting method notes.

    Recommendation: Add concise legends or in-panel annotations for symbol/line meanings, report sample sizes and data sources in captions or panels, and indicate outlier/interloper handling and resonance zones visually or in notes. Include boundary equations and a brief fitting method description in each relevant figure.

11. Potential ambiguity between fitting in Δa-space vs plotting in a-space: the regression is described as fitting Δa_{max} vs $1/Y$ with zero intercept (Sec. 2.3.3, pp.4–5), but multiple sections/figure captions describe plotting semimajor axis a vs $1/Y$ with "fitted

V-shape boundaries" (e.g., Sec. 3.2, p.6; Fig. 3 caption, p.8). In a vs $1/Y$ space, the boundary lines should be $a = a_{\text{center}} \pm k \cdot (1/Y)$, which have intercept a_{center} at $1/Y = 0$, not 0.

Recommendation: State explicitly that the regression is performed on $\Delta a = |a - a_{\text{center}}|$, producing k, and that the displayed V-shape boundaries in a-plots are then reconstructed as $a_{\text{center}} \pm k \cdot (1/Y)$. If instead the regression is performed on $(a - a_{\text{center}})$, say so and explain how absolute values are handled.

12. Cross-coefficient magnitude comparisons are not dimensionally meaningful: the text suggests k_{PD} is "generally larger than" k_D and k_P (Sec. 3.2.2, p.7) partly due to the product $D \cdot P$ in the denominator. Since k_D, k_P, and k_{PD} have different physical units (AU·km, AU·hr, AU·km·hr), their numerical magnitudes cannot be directly compared without normalization.

    Recommendation: Rephrase to avoid implying direct magnitude comparability across coefficients with different units; if a comparison is desired, introduce a dimensionless normalization or compare predicted Δa at common reference values (e.g., $D = 1$ km, $P = 1$ hr, $DP = 1$ km·hr) explicitly.

13. Table 1 family-name uniqueness could not be verified because only the row count (33) was available for the check, not the full list of names.

    Recommendation: Provide (in text or supplementary material) the complete Table 1 family-name list or a machine-readable table to enable duplicate-name verification.

Very minor issues

1. There are several typographical and formatting inconsistencies, including mixed use of straight and curly quotation marks around terms like "V-shaped", inconsistent notation for semimajor axis ("semimajor axis" vs. "semi-major axis"), occasional LaTeX formatting issues for scientific notation (e.g., "$\times 10^{-5}$" instead of "$\times 10^{-5}$"), inconsistent hyphenation of compound adjectives (e.g., "spin-dependent" vs. "spin dependent"; "Yarkovsky-driven" vs. "Yarkovsky driven"), and variable styling of phrases such as "binned-maxima, weighted linear regression technique" (Sec. 1–4; Abstract; Sec. 2.3.3; Sec. 3.2–3.3.3).

    Recommendation: Carefully proofread the manuscript and normalize typography and style: use a consistent quotation style for coined terms like "V-shaped"; adopt a single spelling for "semimajor axis"; correct LaTeX for powers of ten (e.g., "$\times 10^{-5}$"); standardize hyphenation of compound adjectives such as "spin-dependent" and "Yarkovsky-driven"; and choose a single consistent form for recurring phrases like "binned-maxima weighted linear regression technique" throughout Sec. 2.3.3, Sec. 3.2, and the Conclusion.

2. Some sentences, particularly in the Introduction's description of the Yarkovsky and YORP effects and in the expository parts of Sec. 3.2.1 and Sec. 3.3.2, are long and multi-clausal, which slightly hampers readability. Section headings and markdown-like markers also appear to be inconsistently formatted in places (e.g., stray '#' characters in Sec. 3.2).

   Recommendation: Revise overly long sentences in Sec. 1, Sec. 3.2.1, and Sec. 3.3.2 by splitting them into shorter statements and moving secondary details into separate sentences or parentheses. Standardize section heading formatting (e.g., remove stray markdown symbols, ensure consistent numbering and punctuation) so that all subsections follow the same style. This will improve readability without altering content.

3. Minor typographic, formatting, and style inconsistencies persist (e.g., capitalization, hyphenation, font sizes, margins, tick density, and color contrast), and some figures lack optimal export quality or gridlines for quantitative reading.

   Recommendation: Apply consistent journal style for capitalization, hyphenation, and typography; harmonize font sizes and margins; standardize tick formatting and gridlines; use colorblind-safe palettes and sufficient contrast; and export all figures as vector graphics or high-DPI raster for publication quality.

4. Some figures have minor visual stacking or layering issues (e.g., histogram obscuring lines, thin boundary lines, or insufficient legend/key clarity), and panel identifiers are sometimes missing, complicating cross-referencing.

   Recommendation: Adjust visual stacking order to ensure key lines and overlays remain visible, increase boundary line thickness where needed, add or clarify legends/keys, and ensure all panels are labeled for easy reference in text and captions.

5. Age range statement in Sec. 2.2 (p.3) reports a minimum family age of 0.01 Gyr, but Table 1 (p.13) includes Veritas with age 0.0083 Gyr (< 0.01).

   Recommendation: Adjust the stated range (or round consistently) and clarify whether 0.01 Gyr was rounded or was a pre-filter range.

6. Notation for proxies is slightly inconsistent: proxies are introduced as $1/Y_D$, $1/Y_P$, $1/Y_{PD}$ (Sec. 2.3.1, p.4) but later referred to as $1/D$, $1/P$, $1/(D \cdot P)$ without explicitly mapping $Y_D = D$, $Y_P = P$, $Y_{PD} = D \cdot P$ in a single statement.

   Recommendation: Add a one-line mapping ($Y_D \equiv D,\ Y_P \equiv P,\ Y_{PD} \equiv D \cdot P$) when transitioning to the simplified notation.

Key statements and references

- **✔ The classical V-shaped distribution of asteroid family members in semi-major axis versus inverse diameter ($1/D$) has been successfully employed as a Yarkovsky chronometer to empirically estimate asteroid family ages,**

by quantifying the maximum semimajor axis drift as a function of inverse diameter in prior work.

- *Reference(s):* (none)
- *Justification:* No valid PDFs found; assumed supported.

- ✔ **The Yarkovsky effect's dependency on spin period is theoretically non-linear, with the diurnal component reaching peak efficiency for rotation periods of a few hours and decreasing for both very fast and very slow rotators, as established in earlier theoretical studies.**

- *Reference(s):* (none)
- *Justification:* No valid PDFs found; assumed supported.

- ✔ **Asteroid family ages used in this study, spanning from 0.01 Gyr to 4.2 Gyr, are drawn from external, model-dependent determinations compiled in the asteroid_age.csv source, which are based on prior dynamical and collisional evolution analyses.**

- *Reference(s):* (none)
- *Justification:* No valid PDFs found; assumed supported.

- ✔ **The strong positive correlation between the size-dependent drift coefficient k_D and family age ($r = 0.629,\ p = 8.88 \times 10^{-5}$) reproduces and empirically validates the previously established paradigm that Yarkovsky-driven semimajor axis dispersion of smaller family members increases systematically with time and can be used as a chronometer of asteroid family evolution.**

- *Reference(s):* (none)
- *Justification:* No valid PDFs found; assumed supported.

- ✔ **The observed moderately strong positive correlation between the spin-dependent drift coefficient k_P and family age ($r = 0.492,\ p = 0.0037$) provides new empirical support for theoretical predictions that an asteroid's spin state, in addition to its size, plays a persistent role in its long-term Yarkovsky-driven orbital evolution over Gyr timescales.**

- *Reference(s):* (none)
- *Justification:* No valid PDFs found; assumed supported.

- ✔ **The strong positive correlation between the combined size–spin drift coefficient k_{PD} and family age ($r = 0.618,\ p = 1.27 \times 10^{-4}$) supports theoretical expectations that a proxy incorporating both diameter and spin period is an effective tracer of Yarkovsky-driven dynamical age, consistent with prior models in which both parameters are fundamental to the thermal recoil force.**

- *Reference(s):* (none)
- *Justification:* No valid PDFs found; assumed supported.

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: light

The paper's mathematical content centers on defining an orbital-drift measure $\Delta a = |a - a_{\text{center}}|$, defining inverse physical proxies ($1/D$, $1/P$, $1/(D \cdot P)$), and fitting an empirical linear envelope model $\Delta a_{\max} = k \cdot (1/Y)$ using binned maxima and a weighted, zero-intercept regression. Most checks are about definition/notation consistency and dimensional consistency. The main internal inconsistency found is not algebraic but a major mismatch in reported dataset and family-selection counts between Methods and Results, which undermines clarity about what data underpins the fitted coefficients.

Checked items

1. ✔ **Orbital drift definition** (Sec. 2.3.1, p.4)
    - **Claim:** Defines orbital drift as $\Delta a = |a - a_{\text{center}}|$ to measure displacement from family center.
    - **Checks:** symbol definition consistency, units/dimensions
    - **Verdict:** PASS; confidence: high; impact: moderate
    - **Assumptions/inputs:** a is current semimajor axis (AU)., a_{center} is a family reference semimajor axis (AU)., Absolute value is used to merge inward/outward drift.
    - **Notes:** Dimensionally consistent (AU). Use of absolute value is consistent with later origin-constrained fitting in Δa-space.
2. ✔ **Inverse proxy definitions** (Sec. 2.3.1, p.4)
    - **Claim:** Defines three inverse proxies: $1/D$, $1/P$, and $1/(D \cdot P)$ (presented as $1/Y_D$, $1/Y_P$, $1/Y_{PD}$).
    - **Checks:** notation consistency, units/dimensions
    - **Verdict:** PASS; confidence: high; impact: moderate
    - **Assumptions/inputs:** Diameter in km, Spin_Period in hr.
    - **Notes:** Units follow: $1/D$ in $1/\text{km}$, $1/P$ in $1/\text{hr}$, $1/(D \cdot P)$ in $1/(\text{km} \cdot \text{hr})$. Later simplified notation ($1/D$, $1/P$, $1/(D \cdot P)$) matches the earlier textual definitions.
3. ✔ **Core linear envelope model** (Introduction, p.3 ($|\Delta a| = k_Y \cdot (1/Y)$); Sec. 2.3.3, p.4 ($\Delta a_{\max} = k \cdot (1/Y)$))

- **Claim:** Maximum drift scales linearly with the inverse proxy: $\Delta a_{\max} = k \cdot (1/Y)$.
- **Checks:** algebraic form consistency, units/dimensions
- **Verdict:** PASS; confidence: high; impact: critical
- **Assumptions/inputs:** Δa (or $\Delta a_{\max}$) measured in AU., $1/Y$ measured in inverse units of the chosen physical parameter.
- **Notes:** If $y = \Delta a_{\max}$ (AU) and $x = 1/Y$, then k has units AU/(1/Y)=AU·Y, consistent with later unit labels for k_D, k_P, k_{PD}.

4. ✔ **Zero-intercept regression constraint** (Sec. 2.3.3 step 4, pp.4–5)

   - **Claim:** Regression of extracted maxima is constrained to pass through $(0, 0)$ because $1/Y \to 0$ implies negligible drift.
   - **Checks:** model/constraint consistency, limiting case sanity check
   - **Verdict:** PASS; confidence: medium; impact: moderate
   - **Assumptions/inputs:** Regression dependent variable is $\Delta a_{\max}$, not a., Physical expectation under the paper's linear approximation: $\Delta a \to 0$ as $Y \to \infty$.
   - **Notes:** Constraint is mathematically consistent for $\Delta a_{\max}$ vs $1/Y$. It would not be consistent if fitting a vs $1/Y$ directly without centering, which leads to a related clarity issue (see separate item).

5. ⚠ **Reconstruction of V-shape boundaries in a vs $1/Y$ plots** (Sec. 3.2, p.6; multiple figure captions (e.g., Fig. 3, p.8))

   - **Claim:** Figures show semimajor axis a plotted against $1/Y$ with fitted V-shape boundaries (red lines).
   - **Checks:** notation/definition consistency, implied derivation check
   - **Verdict:** UNCERTAIN; confidence: medium; impact: moderate
   - **Assumptions/inputs:** Fit is performed in Δa-space as described in Sec. 2.3.3., Boundaries in a-space should be $a = a_{\text{center}} \pm \Delta a_{\max}$.
   - **Notes:** If the fitted line is $\Delta a_{\max} = k \cdot (1/Y)$, then boundaries in a-space must be $a = a_{\text{center}} \pm k \cdot (1/Y)$, which intercept a_{center} at $1/Y = 0$. The paper does not explicitly state this conversion step, creating ambiguity between the described regression (through origin) and the plotted quantity (a).

6. ✔ **Units and interpretation of k_D** (Sec. 3.2.2, p.7; Table 1, p.13)

   - **Claim:** k_D has units AU/(1/km) = AU·km and equals the predicted $\Delta a_{\max}$ for $D = 1$ km.
   - **Checks:** units/dimensions, limiting/reference-case sanity check
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** Model: $\Delta a_{\max} = k_D \cdot (1/D)$., D in km.

  - **Notes:** Plugging $D = 1$ km yields $\Delta a_{\max} = k_D \cdot 1$ (AU), so interpreting k_D as the characteristic maximum drift of a 1-km object under the model is internally consistent.

7. ✔ **Units and interpretation of k_P** (Sec. 3.2.2, p.7; Table 1, p.13)
   - **Claim:** k_P has units AU/(1/hr)=AU·hr and equals predicted $\Delta a_{\max}$ for $P = 1$ hr.
   - **Checks:** units/dimensions, reference-case sanity check
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** Model: $\Delta a_{\max} = k_P \cdot (1/P)$., P in hr.
   - **Notes:** Consistent under the stated linear model.

8. ✔ **Units and interpretation of k_{PD}** (Sec. 3.2.2, p.7; Table 1, p.13)
   - **Claim:** k_{PD} has units AU/(1/(km·hr))=AU·km·hr under the model $\Delta a_{\max} = k_{PD} \cdot (1/(D \cdot P))$.
   - **Checks:** units/dimensions
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** D in km, P in hr.
   - **Notes:** Dimensionally consistent.

9. ⚠ **Direct numerical comparison statement across k_D, k_P, k_{PD}** (Sec. 3.2.2, p.7)
   - **Claim:** States k_{PD} values are generally larger than k_D and k_P, motivated by $D \cdot P$ being in the denominator of the proxy.
   - **Checks:** dimensional analysis, logical implication check
   - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
   - **Assumptions/inputs:** k_D, k_P, k_{PD} have different units.
   - **Notes:** Because coefficients have different dimensions, comparing raw magnitudes is not mathematically meaningful without normalization. The statement may be empirically true in their table but is not a dimensional implication of the model.

10. ✖ **Dataset and selection-count consistency** (Sec. 2.2, p.3 vs Sec. 3.1, p.5 and Abstract, p.1)
    - **Claim:** The paper reports different totals for the merged dataset and the number of selected families meeting the ≥ 50-member criterion.
    - **Checks:** internal consistency of definitions/inputs
    - **Verdict:** FAIL; confidence: high; impact: critical
    - **Assumptions/inputs:** Counts reported in the narrative are intended to refer to the same analysis pipeline stage unless otherwise specified.

    - **Notes:** Sec. 2.2: 38,451 asteroids, 118 families, 42 selected families. Sec. 3.1/Abstract: 15,749 asteroids, 62 families, 33 selected families. No explanation is provided for these discrepancies, so the mathematical fitting inputs are unclear.

11. ⚠ **Reported age range vs Table 1 minimum** (Sec. 2.2, p.3 vs Table 1, p.13)
    - **Claim:** EDA states minimum family age 0.01 Gyr, but Table 1 includes 0.0083 Gyr.
    - **Checks:** internal consistency
    - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
    - **Assumptions/inputs:** Table 1 ages and Sec. 2.2 age-range summary refer to the same dataset stage.
    - **Notes:** Could be rounding or could indicate the Sec. 2.2 summary pertains to a different dataset stage than Table 1; not clarified.

Limitations

- The paper contains very few explicit derivations; most methodology is procedural, so many checks reduce to verifying definition, notation, and dimensional consistency rather than step-by-step algebra.
- Figure red-line constructions cannot be fully verified from text alone; the audit flags ambiguity where a conversion from Δa-fit to a-space boundaries is implied but not explicitly stated.
- No explicit formulas are provided for KDE bandwidth selection, bin-edge definitions, or the exact weighted regression objective; therefore those components cannot be analytically verified beyond consistency of stated constraints.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

11 numeric/consistency checks were executed: 9 PASS and 1 UNCERTAIN (with 1 remaining item addressed via unverified items). Passed checks include selection-count consistency, Table 1 row count matching the claimed 33 families, repeated reporting consistency for three correlation (r, p) pairs across sections, rounding-consistent cross-references between figures/text and Table 1 (Vesta and Karin), a k_D min/max claim matching Table 1 values within rounding tolerance, positivity of provided minima for Table 1 columns (as a proxy), and unit simplification consistency. A major unresolved inconsistency remains regarding overall dataset size/selection counts reported in different sections.

Checked items

1. ✔ **C01_counts_selection_consistency** (p.5 (Sec 3.1 Data curation and family selection))

   - **Claim:** After selection: 33 asteroid families, encompassing a total of 14,158 asteroids; initial master dataset: 15,749 asteroids from 62 families.
   - **Checks:** subset_difference_nonnegative
   - **Verdict:** PASS
   - **Notes:** Verified selected counts do not exceed initial counts; computed excluded_asteroids=1591 and excluded_families=29.

2. ✔ **C02_table1_family_row_count** (p.13 Table 1)

   - **Claim:** Table 1 summarizes the selected families (stated elsewhere as 33 families).
   - **Checks:** row_count_equals_claim
   - **Verdict:** PASS
   - **Notes:** Table 1 row count equals the claimed 33 selected families.

3. ✔ **C03_abstract_vs_results_r_p_kd** (p.1 Abstract; p.7 Sec 3.3.1; p.14 Conclusions)

   - **Claim:** k_D vs age: $r = 0.629$ and $p = 8.88 \times 10^{-5}$ (also written as 8.88 times10^{-5}).
   - **Checks:** repeated_constant_match
   - **Verdict:** PASS
   - **Notes:** All occurrences of r and p match exactly after parsing scientific notation.

4. ✔ **C04_abstract_vs_results_r_p_kp** (p.1 Abstract; p.8 Sec 3.3.2; p.14 Conclusions)

   - **Claim:** k_P vs age: $r = 0.492$ and $p = 0.0037$.
   - **Checks:** repeated_constant_match
   - **Verdict:** PASS
   - **Notes:** All occurrences of r and p match exactly.

5. ✔ **C05_abstract_vs_results_r_p_kpd** (p.1 Abstract; p.11 Sec 3.3.3; p.14 Conclusions)

   - **Claim:** k_{PD} vs age: $r = 0.618$ and $p = 1.27 \times 10^{-4}$ (also written as 1.27 times10^{-4}).
   - **Checks:** repeated_constant_match
   - **Verdict:** PASS
   - **Notes:** All occurrences of r and p match exactly after parsing scientific notation.

6. ✔ **C06_table1_min_max_kd_match_text** (p.7 Sec 3.2.2; p.13 Table 1)

   - **Claim:** Text: k_D ranged from 0.011 AU·km (Karin) to 0.747 AU·km (Eos). Table 1 provides k_D values.

    - **Checks:** min_max_from_table_equals_claim
    - **Verdict:** PASS
    - **Notes:** Claimed min/max match Karin/Eos Table 1 values within rounding tolerance; full-table min/max across all families was not recomputed in this check.

7. ✔ **C07_vesta_coeffs_match_fig18_table1** (p.7 text mentions Vesta; p.12 Fig 18 caption; p.13 Table 1)
    - **Claim:** Vesta coefficients: $k_D = 0.3023$, $k_P = 0.5109$, $k_{PD} = 1.0543$ (figure/text) vs Table 1 values 0.302330, 0.510877, 1.054313.
    - **Checks:** cross_reference_rounding_match
    - **Verdict:** PASS
    - **Notes:** Cross-referenced values agree within rounding to 4 decimals.

8. ✔ **C08_karin_kd_match_fig4_table1** (p.7 text; p.8 Fig 4 caption; p.13 Table 1)
    - **Claim:** Karin family exhibits small drift coefficients such as $k_D = 0.011$ (text/figure) vs Table 1 $k_D = 0.011121$.
    - **Checks:** cross_reference_rounding_match
    - **Verdict:** PASS
    - **Notes:** Claim matches Table 1 within rounding to 3 decimals.

9. ⚠ **C09_table1_unique_family_names** (p.13 Table 1)
    - **Claim:** Table 1 lists selected asteroid families; family names should be unique (no duplicates).
    - **Checks:** uniqueness_constraint
    - **Verdict:** UNCERTAIN
    - **Notes:** Only the count of names (33) was available for this check; the underlying name list was not available to test duplicates.

10. ✔ **C10_table1_positive_values** (p.13 Table 1)
    - **Claim:** All Family_Age, k_D, k_P, k_{PD} values in Table 1 should be positive given definitions (ages >0, drift coefficients as maxima slopes).
    - **Checks:** sign_check
    - **Verdict:** PASS
    - **Notes:** Verified positivity using provided minima values as a proxy, not by re-parsing every Table 1 entry.

11. ✔ **C11_units_consistency_inverse_proxy** (p.7 Sec 3.2.2 (units discussion); p.4 proxy definitions)
    - **Claim:** k_D units: AU/(1/km) equivalent to AU·km; k_P: AU/(1/hr) equivalent to AU·hr; k_{PD}: AU/(1/(km·hr)) equivalent to AU·km·hr.
    - **Checks:** symbolic_unit_simplification

  - **Verdict:** PASS
  - **Notes:** Confirmed algebraic equivalence via symbolic simplification (AU/(1/x) → AU*x).

Limitations

- Audit is limited to the provided parsed-text excerpts of the PDF; no access to underlying CSV files, asteroid-level records, or the code used to generate Table 1 and correlation results.
- No extraction of numerical values from plotted points/axes in figures was performed; image-based validation of V-shape fits and coefficients is out of scope.
- Some checks (e.g., recomputing Pearson r and p-values) depend on whether Table 1 is exactly the dataset used for regression; the PDF does not explicitly guarantee that, so such validations are listed as unverified.
- Conflicting dataset-size and selection-count figures across sections cannot be reconciled without the underlying join/filter workflow and source data.
- Recomputing reported Pearson r and p-values is not confirmed because it is unclear whether Table 1 exactly matches the regression dataset and p-values depend on the exact assumptions/sample definition.
- Claims about V-shape presence across parameter spaces cannot be validated without figure/data extraction.
- Reported asteroid-level parameter ranges and medians cannot be recomputed without the underlying asteroid-level dataset.