Skepthical review: *Renormalization Group Analysis of PINN Latent Space Structure for the 2D Burger's Equation*

Summary

This manuscript proposes an RG-inspired, multi-scale analysis of how a Physics-Informed Neural Network (PINN) represents solutions of a (purported) 2D Burgers problem in a 10D latent space across 25 viscosity values. Latent vectors evaluated on a structured spatiotemporal grid are iteratively coarse-grained via non-overlapping 2×2 block averaging (Sec. 2.2), treated as an RG "scale" index s. At each (ν, s), the authors standardize latent coordinates (Sec. 2.3), perform PCA, and track eigenvalue spectra, an effective dimensionality $\mathbf{ED}_{99}$ (Sec. 2.4.2), and a normalized Shannon entropy of the eigenvalue distribution (Sec. 2.4.3), interpreting their evolution with s as an "RG flow" (Sec. 2.5). The direction is timely—connecting multiscale physics intuition with representation analysis for PINNs—but the current draft is not yet scientifically reviewable: Sec. 3 contains placeholder figures/text and therefore does not substantiate the claimed viscosity-dependent flow behaviors stated in the Abstract/Introduction/Conclusions. In addition, the PDE/data description is internally inconsistent (Sec. 2.1), and the PINN/training/latent-extraction setup is under-specified, preventing reproducibility and weakening the physical interpretation of the proposed latent-space diagnostics.

Strengths

- Conceptually interesting and timely: an RG/coarse-graining perspective on PINN latent representations across physical regimes controlled by viscosity (Introduction; Sec. 2.2–2.5).
- The pipeline components—iterative 2×2 spatiotemporal block averaging, standardization, PCA, eigenvalue-based $\mathbf{ED}_{99}$, and entropy—are mostly clearly laid out and could be reproducible once missing implementation details are added (Sec. 2.2–2.4).
- Using multiple complementary diagnostics (spectra, $\mathbf{ED}_{99}$, entropy) across many viscosities has the potential to yield a useful "representation phase diagram" over (ν, s) if results and uncertainty are properly reported (Sec. 2.4–2.5).
- The eigenvalue normalization $p_k = \lambda_k / \sum_j \lambda_j$ enabling Shannon entropy is mathematically coherent and interpretable as spectral uniformity (Sec. 2.4.3).

Major issues

1. **Sec. 3 is incomplete (placeholders for figures/captions and generic filler text), so none of the central empirical claims (e.g., viscosity-dependent entropy/ED_{99} trends; approach to fixed-point-like behavior) stated in the Abstract, Introduction, and Conclusions can be verified (Sec. 1, Sec. 3, Sec. 4).**

Recommendation: Populate Sec. 3 with the actual experimental results and make every key claim traceable to a figure/table: (i) plots of \$\mathrm{ED}*{99}(s) andnormalizedentropy(s) forrepresentativeviscositiesspanningclearlydefinedregimes*; (*ii*)*heatmapsover (\nu, s)for \mathrm{ED} andentropy*; (*iii*)*representativePCAeigenvaluespectraatselected*(\nu, s) .*Includeaxisdefinitions*, *scaleindex*s *meaning*, *andnumericalsummaries*(*e.g.*, *large* − *scalevalues*, *slopesvs*s\$, crossover scales). Then revise Abstract/Sec. 1/Sec. 4 to cite specific figures and quantify the reported trends rather than stating them qualitatively without evidence.

2. **The governing PDE, variables, and dataset geometry are internally inconsistent: Sec. 2.1 writes a PDE with x and y derivatives and an advecting field v ($\partial u/\partial t + u\,\partial u/\partial x + v\,\partial u/\partial y = \nu(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2)$), but the dataset/analysis is defined on an (x, t) grid only ($N_x \times N_t = 101 \times 103$) with inputs (x, t, ν) and no y dimension or definition of v. This prevents interpreting what physical system the latent vectors correspond to (Sec. 2.1).**

   Recommendation: Make the PDE and data tensor dimensions consistent, explicitly and unambiguously: either (a) revise to 1D-in-space Burgers (x, t) and remove y and v everywhere; or (b) present the full 2D-in-space Burgers system (typically coupled u and v equations), include y (and N_y) in the dataset description and coarse-graining, and specify how u/v are represented in the latent data. In either case, fully specify domain, boundary/initial conditions, and the viscosity set (list or range) in Sec. 2.1, and ensure the notation matches the actual grid used in Sec. 2.2–2.4.

3. **The PINN and latent extraction are under-specified, undermining reproducibility and interpretation. The manuscript does not state whether there is one conditional PINN over ν or 25 separate models; where exactly the "10D latent vector" is taken (which layer; pre-/post-activation); training details (loss terms/weights, collocation and boundary sampling, optimizer, epochs); and baseline solution accuracy versus a reference solver. Representation geometry can depend strongly on these choices (Sec. 2.1).**

   Recommendation: Expand Sec. 2.1 (or add a dedicated 'PINN and data generation' subsection) to include: architecture (layers/widths/activations), how ν enters the network, definition/location of the 10D latent, whether the latent is shared/comparable across viscosities (single model) or model-specific (multiple models), full loss decomposition and weighting, training protocol and sampling counts, and quantitative accuracy metrics (e.g., relative L_2 error vs numerical reference) across ν. If reusing a prior model/dataset (Auddy et al., 2023; Baty, 2024), identify the exact checkpoint/version and provide access information (repo/DOI) and any modifications.

4. **The ‘RG flow/fixed point/attractor’ framing is currently qualitative and risks over-claiming. The transformation used is block averaging of latent vectors plus per-scale re-standardization before PCA; there is no explicit rescaling step, no parameter/coupling flow, and per-scale z-scoring can remove amplitude information and change how ‘flow’ should be interpreted (Sec. 2.2–2.5; Sec. 4).**

   Recommendation: In Sec. 2.5 and Sec. 4, tighten the conceptual claims: clearly state what aspects are RG-inspired rather than a formal RG. Provide operational definitions (e.g., what constitutes a ‘fixed point’: stabilization of ED_{99}/entropy/spectrum within a tolerance over successive s). Justify why coarse-graining latents (rather than physical fields) is meaningful for ‘integrating out’ small scales. Add at least one baseline comparison in Sec. 3 to show the value of tracking multi-step flows (e.g., metrics on a single downsampled scale; or downsampling the latent field without iterative flow) and comment on the effect of per-scale standardization (e.g., repeat key plots with global standardization fixed at $s = 0$).

5. **Uncertainty/finite-sample effects are not addressed, even though sample size shrinks rapidly with coarse-graining, making PCA eigenvalues, entropy, and ED_{99} potentially unstable at large s. The manuscript also does not report the actual number of RG steps used and $N^{(s)}$ at each step for the 101×103 grid (Sec. 2.2; Sec. 3).**

   Recommendation: In Sec. 2.2, tabulate $N_x^{(s)}, N_t^{(s)}$, and $N^{(s)}$ for $s = 0, 1, 2, \ldots$ until termination for $N_x = 101,\ N_t = 103$ with $b_x = b_t = 2$, and state the maximum s retained (and the criterion $N^{(s)} \geq 20$). In Sec. 3, add uncertainty estimates: bootstrap or subsample grid points at each (ν, s) to put confidence intervals/error bars on eigenvalues, entropy, and ED_{99}. Clearly mark in plots where $N^{(s)}$ becomes small and restrict ‘fixed point’ interpretation to scales with demonstrably stable estimates.

6. **Robustness to design choices (block size, ED threshold, latent dimension, standardization choice, edge handling) is not tested, so regime-dependent conclusions may be artifacts of specific settings (Sec. 2.2–2.4; Sec. 4).**

   Recommendation: Add a compact sensitivity study in Sec. 3: (i) compare 2×2 with at least one alternative coarse-graining (e.g., 3×3, or anisotropic $2 \times 1/1 \times 2$) and report whether qualitative ν-dependent trends persist; (ii) report ED at multiple thresholds (e.g., \$\mathrm{ED}*{95}inadditionto\mathrm{ED}*)*and/oranalternativeeffective* − *rankmetric*(*participationratio*); (*iii*)*explicitlycompareper* − (\nu,s) *standardizationvsglobal*/*per* − \nu\$ standardization (Sec. 2.3); (iv) justify the 10D latent choice and, if feasible, show at least one alternate latent size or clearly state it as a limitation.

7. **The physical interpretation (latent 'complexity' tracking advection- vs diffusion-dominated regimes) is not supported by direct comparisons to physical-field diagnostics, making it unclear whether latent entropy/ED_{99} changes reflect actual solution multiscale structure or representation idiosyncrasies (Sec. 2.5; Sec. 4).**

   Recommendation: In Sec. 3, add a small set of physics-side diagnostics computed from the PINN solution (or reference solution) and relate them to latent metrics: e.g., gradient magnitude statistics/total variation/shock indicators/correlation length proxies versus ν and s. Include at least qualitative side-by-side plots (coarse-grained physical fields and latent metrics) for a few viscosities. If you claim a regime transition, define it numerically (ν ranges or a Reynolds-like proxy) and show where the transition appears in both physical and latent measures.

Minor issues

1. Viscosity regimes ('low/intermediate/high') are used throughout without explicit numerical definitions, and there are no group-wise summaries across ν despite the paper's main framing around regime dependence (Sec. 1; Sec. 2.1; Sec. 4).

   Recommendation: Define viscosity regimes quantitatively in Sec. 2.1 or Sec. 3 (specific ν ranges and/or a dimensionless proxy). In Sec. 3, report aggregate statistics within regimes (means/SDs of ED_{99} and entropy at selected scales; distribution across the 25 viscosities), and use those definitions consistently in Sec. 4.

2. Metric motivation and limitations are under-discussed: ED_{99} in a 10D latent space can saturate; entropy is sensitive to sampling noise and preprocessing; and the 'complexity' interpretation is asserted more than justified (Sec. 2.4.2–2.4.3; Sec. 2.5).

   Recommendation: In Sec. 2.4.2–2.4.3, add brief motivation and known pitfalls (finite-sample bias, saturation in low latent dimension). Consider adding one additional spectrum inequality metric (e.g., participation ratio/effective rank/Gini) in Sec. 3 to demonstrate that conclusions do not depend on ED_{99} and Shannon entropy alone.

3. Standardization is described but not fully specified (computed per (ν, s) vs pooled across ν and/or s), and this choice materially affects cross-ν and cross-scale comparability (Sec. 2.3; Sec. 2.5).

   Recommendation: Clarify the standardization scope in Sec. 2.3 and justify it. In Sec. 3, include a brief comparison showing how key trends change (or remain robust) under an alternative standardization strategy.

4. The coarse-graining procedure's boundary handling is not clearly stated: floor operations imply dropping leftover rows/columns when N_x, N_t are odd (101, 103), but which points are discarded (or whether padding is used) is unspecified (Sec. 2.2).

Recommendation: State explicitly how remainders are handled (drop last index; pad; smaller boundary blocks; etc.). If dropping, quantify how many points are discarded per scale and briefly comment on whether this could bias results (especially for structured dynamics).

5. The manuscript would benefit from clearer separation of Results vs Interpretation/Limitations; RG interpretation and broader claims are interwoven with summary statements (Sec. 4).

   Recommendation: Consider adding a short Discussion section between Sec. 3 and Sec. 4 to interpret the observed flow patterns, relate them to physics and baselines, and list limitations (single PDE, single architecture, heuristic RG analogy, sensitivity). Keep Sec. 4 as a concise conclusions/outlook.

6. Data/model availability is not clearly stated, despite dependence on latent outputs (Sec. 2.1).

   Recommendation: Add a Data/Code availability statement: where to access latent tensors (or how to regenerate them), exact viscosity list, and scripts for coarse-graining/PCA/plotting.

7. Some references appear tangential to the core PINN/RG-in-ML topic, and the bibliography has formatting inconsistencies (References).

   Recommendation: Tighten citations toward directly relevant work (PINN representation analysis, multiscale/renormalization perspectives in ML) and standardize the reference style; remove duplicates/malformed entries.

Very minor issues

1. Sec. 2.2 coarse-graining summation notation is hard to parse and appears malformed (ambiguous index limits), preventing a strict check that the operation averages exactly $b_x \cdot b_t$ points per block.

   Recommendation: Rewrite the coarse-graining equation with explicit index ranges (e.g., j from $j'b_x$ to $(j'+1)b_x - 1$; k from $k'b_t$ to $(k'+1)b_t - 1$) and define all indices. Add a short note on implementation for odd sizes.

2. Entropy definition: the log base is not specified, and normalization by $\log(10)$ only guarantees $[0, 1]$ when using the same base consistently; also the small constant inside log should be clearly labeled as numerical stabilization (Sec. 2.4.3).

   Recommendation: Specify log base (e.g., natural log) and use the same base for normalization. Explicitly state that the $+1\mathrm{e}{-9}$ term is for numerical stability.

3. Standardization mentions adding ϵ to avoid division by zero, but the displayed formula omits ϵ (Sec. 2.3).

Recommendation: Include ϵ in the equation $(\sigma_d + \epsilon)$ or state clearly it is an implementation detail not shown in the math.

4. PCA operator details are not fully pinned down (covariance vs correlation matrix; SVD convention), which affects the precise definition/units of λ_k (Sec. 2.3).

   Recommendation: State the exact PCA computation (e.g., eigenvalues of $(1/(N-1))Z^{\mathrm{T}}Z$ for standardized Z, or SVD of Z) and confirm λ_k are proportional to explained variance.

5. Presentation/formatting issues: placeholder figure captions/text in Sec. 3; minor typos (split words), inconsistent hyphenation ('spatial-temporal' vs 'spatiotemporal'), and inconsistent notation/indexing (Secs. 1–4).

   Recommendation: Remove all placeholders; proofread for typographical consistency; standardize terminology ('spatiotemporal'), indexing conventions, and symbol formatting (e.g., ED_{99}).

6. Manuscript metadata/formatting includes unusual author/institution lines (e.g., tool/server mentions) that are not standard for scientific submissions.

   Recommendation: Replace with standard author/affiliation formatting appropriate for the target venue and move any tooling acknowledgments to an Acknowledgments section if needed.

Key statements and references

- • **Principal Component Analysis (PCA) is used to quantify the structure and complexity of the PINN latent space at each coarse-graining scale, following established applications of PCA to astrophysical and cosmological data analysis, including its use to explore star formation histories and high-resolution spectroscopic data, and to study the impact of the cosmic web on galaxy properties.**
- *Reference(s):* Ferreras et al., 2006, Damiano et al., 2019, Nandi and Pandey, 2025
- • **The effective dimensionality metric ED_{99}, defined as the minimum number of principal components required to explain at least 99% of the total variance via the cumulative explained variance ratio, follows prior work that employs PCA-based dimensionality reduction for scientific inference and representation analysis in astronomy and cosmology.**
- *Reference(s):* Pat et al., 2022, Fan, 2024, Park et al., 2025
- • **The normalized Shannon entropy of the eigenvalue spectrum, defined as $H_{\text{normalized}} = H^{(s)}/\log(10)$ with $H^{(s)} = -\sum_{k=1}^{10} p_k^{(s)} \log(p_k^{(s)} + 10^{-9})$ and $p_k^{(s)} = \lambda_k^{(s)}/\sum_{j=1}^{10} \lambda_j^{(s)}$, is adopted as a measure of how uniformly variance is distrib-**

uted across latent dimensions, extending earlier uses of Shannon entropy to test cosmic homogeneity and isotropy and to quantify information content in galaxy spectra and dynamical systems.

- *Reference(s):* Pandey, 2013, Pandey, 2016, Ferreras et al., 2023

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: substantial

The paper's analytic core is the definition of an RG-like coarse-graining operator on latent vectors defined on a spatiotemporal grid, followed by PCA-based spectral metrics (explained variance, ED_{99}, Shannon entropy) tracked across scales and viscosities. Most metric definitions are internally compatible, but there is a critical inconsistency between the stated governing PDE (involving x, y and an undefined v field) and the dataset/analysis (using only x, t, ν). Additionally, the displayed coarse-graining summation is not written in a verifiable form.

Checked items

1. ✖ **Burgers PDE vs stated problem dimensionality** (Sec. 2.1, p. 2 (displayed PDE))
   - **Claim:** Defines the governing "2D Burger's equation" as $\partial u/\partial t + u\,\partial u/\partial x + v\,\partial u/\partial y = \nu(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2)$.
   - **Checks:** symbol definition consistency, dimensional/variable consistency
   - **Verdict:** FAIL; confidence: high; impact: critical
   - **Assumptions/inputs:** u is a scalar field depending on space and time, v is another field or parameter used in advection term, x, y are spatial coordinates and t is time
   - **Notes:** The PDE introduces y and v but neither is defined elsewhere in the methods, and later the data/analysis only uses (x, t, ν). As written, it is not possible to symbolically reconcile the PDE with the dataset tensor dimensions and subsequent coarse-graining on an $N_x \times N_t$ grid.

2. ✖ **Dataset coordinates vs PDE variables** (Sec. 2.1, p. 2 (data tensor and features description))
   - **Claim:** Data features are 3 mesh coordinates (x, t, ν) plus 10 latent components, defined on an $N_x \times N_t$ grid for each ν.
   - **Checks:** definition consistency, dimensional/variable consistency
   - **Verdict:** FAIL; confidence: high; impact: critical
   - **Assumptions/inputs:** If PDE is truly 2D-in-space, data should include y (and typically N_y) and possibly latent vectors at (x, y, t, ν), If PDE is 1D-in-space, PDE should not include y or $v\partial u/\partial y$

- **Notes:** The methods define $L(x_j, t_k, \nu_i)$ and a 4D array $(N_x, N_t, N_\nu, N_{\text{features}})$ with no y dimension. This contradicts the PDE containing y-derivatives and makes the physical interpretation of coarse-graining and "2D" ambiguous.

3. ✔ **Latent vector definition and point count** (Sec. 2.1, p. 2)

   - **Claim:** For each viscosity ν_i, there are $N_x \times N_t$ latent vectors of dimension 10; $N_x \times N_t = 101 \times 103 = 10403$.
   - **Checks:** arithmetic sanity-check (symbolic count), definition consistency
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** $N_x = 101$ and $N_t = 103$ as stated, Each grid point yields one 10D latent vector
   - **Notes:** The point-count computation matches the stated grid sizes, and the latent vector dimensionality (10) matches the '13 features = 3 coords + 10 latent' statement.

4. ⚠ **Coarse-graining averaging operator (index ranges)** (Sec. 2.2, p. 3 (displayed coarse-graining equation))

   - **Claim:** Defines $L^{(s+1)}$ at coarse cell (j', k') as the average of $L^{(s)}$ over a non-overlapping $b_x \times b_t$ block ($b_x = b_t = 2$).
   - **Checks:** algebraic/formula well-posedness, indexing consistency
   - **Verdict:** UNCERTAIN; confidence: medium; impact: critical
   - **Assumptions/inputs:** Non-overlapping blocks partition the grid (possibly with remainder dropped via floor), Each block contains exactly $b_x \cdot b_t$ points
   - **Notes:** The summation notation is not syntactically/semantically clear as written (upper limits appear malformed), so the exact mapping from (j', k') to the set of (j, k) being averaged cannot be verified symbolically. The surrounding prose indicates a standard block average, but the displayed equation needs correction for auditability.

5. ✔ **Coarse-grained grid size update** (Sec. 2.2, p. 3)

   - **Claim:** After coarse-graining by factors b_x, b_t, the grid becomes $N_x^{(s+1)} = \text{floor}(N_x^{(s)}/b_x)$, $N_t^{(s+1)} = \text{floor}(N_t^{(s)}/b_t)$.
   - **Checks:** definition consistency, indexing/shape consistency
   - **Verdict:** PASS; confidence: high; impact: moderate
   - **Assumptions/inputs:** Non-overlapping block aggregation, Remainders (if any) are discarded or otherwise not represented
   - **Notes:** This is consistent with non-overlapping blocking under floor reduction. However, the treatment of leftover rows/columns is unspecified (drop/pad/partial blocks).

6. ✔ **Stopping criterion for iterative coarse-graining** (Sec. 2.2, p. 3)

- **Claim:** Iteration stops when $N_x^{(s)} < b_x$ or $N_t^{(s)} < b_t$ or when total points drop below a threshold (example: 20 points) for robust PCA.
- **Checks:** logical consistency
- **Verdict:** PASS; confidence: medium; impact: minor
- **Assumptions/inputs:** PCA requires $N^{(s)}$ sufficiently larger than feature dimension (10)
- **Notes:** Criterion is logically compatible with the coarse-graining procedure and the need for a sample size exceeding the ambient dimension. Exact threshold choice is not a symbolic issue.

7. ⚠ **Standardization (z-scoring) definition** (Sec. 2.3, p. 3 (standardization equation and text))

   - **Claim:** Standardizes each latent dimension independently via $(Z[p, d] - \mu_d)/\sigma_d$; mentions adding $\epsilon = 10^{-9}$ to avoid division by zero.
   - **Checks:** algebraic correctness, definition consistency
   - **Verdict:** UNCERTAIN; confidence: high; impact: moderate
   - **Assumptions/inputs:** μ_d and σ_d computed across points p at fixed scale s, σ_d may be zero for constant dimensions
   - **Notes:** The displayed formula omits the epsilon that the text claims is added in the denominator. If ϵ is part of the mathematical definition, it should appear explicitly; otherwise, the text should frame it as an implementation safeguard.

8. ⚠ **PCA eigenvalues interpreted as explained variance** (Sec. 2.3, p. 3)

   - **Claim:** Eigenvalues λ_k represent variance explained by each principal component.
   - **Checks:** definition completeness
   - **Verdict:** UNCERTAIN; confidence: medium; impact: moderate
   - **Assumptions/inputs:** PCA performed on covariance (or correlation) matrix of standardized data, or equivalently via SVD, Eigenvalues are nonnegative
   - **Notes:** Interpretation is standard but the paper does not specify the exact PCA operator (covariance normalization factor, covariance vs correlation vs SVD). This affects the precise meaning/scale of λ_k (though ratios are typically invariant to the $(N - 1)$ factor).

9. ✔ **Cumulative explained variance ratio formula** (Sec. 2.4.2, p. 4)

   - **Claim:** Defines $C_k^{(s)} = (\sum_{i=1}^{k} \lambda_i^{(s)})/(\sum_{j=1}^{10} \lambda_j^{(s)})$.
   - **Checks:** algebraic correctness, normalization check
   - **Verdict:** PASS; confidence: high; impact: minor

- **Assumptions/inputs:** Eigenvalues are ordered (typically descending), Total variance is $\sum_{j=1}^{10} \lambda_j$
- **Notes:** The ratio is correctly formed as a cumulative fraction of total variance. Minor typographic ambiguity in the summation formatting does not change the intended meaning.

10. ✔ **ED__99 definition from cumulative variance** (Sec. 2.4.2, p. 4)
    - **Claim:** ED_{99} is the smallest integer K such that $C_K^{(s)} \geq 0.99$.
    - **Checks:** logical consistency
    - **Verdict:** PASS; confidence: high; impact: minor
    - **Assumptions/inputs:** $C_k^{(s)}$ is nondecreasing in k when eigenvalues are nonnegative
    - **Notes:** Definition is internally consistent with the cumulative explained variance ratio.

11. ✔ **Eigenvalue-to-probability mapping** (Sec. 2.4.3, p. 4)
    - **Claim:** Defines $p_k^{(s)} = \lambda_k^{(s)} / \sum_{j=1}^{10} \lambda_j^{(s)}$.
    - **Checks:** normalization/constraints
    - **Verdict:** PASS; confidence: high; impact: moderate
    - **Assumptions/inputs:** $\sum_j \lambda_j^{(s)} > 0$ (non-degenerate data), $\lambda_k^{(s)} \geq 0$
    - **Notes:** Given nonnegative eigenvalues and positive total variance, $p_k^{(s)}$ forms a valid probability distribution summing to 1.

12. ⚠ **Shannon entropy definition and normalization** (Sec. 2.4.3, p. 4)
    - **Claim:** Computes $H(s) = -\sum_k p_k^{(s)} \log(p_k^{(s)} + 1e^{-9})$ and normalized entropy $H_{\text{normalized}} = H / \log(10)$.
    - **Checks:** algebraic correctness, bounds/normalization, definition consistency
    - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
    - **Assumptions/inputs:** Same logarithm base used throughout, $p_k^{(s)}$ sum to 1, Epsilon is for numerical stability when $p_k = 0$
    - **Notes:** With ϵ inside the log, the entropy is a slightly modified Shannon entropy; the claim 'maximum possible entropy is $\log(10)$' is exact only for the unmodified form $-\sum p \log p$. The normalization to $[0, 1]$ also requires consistent log bases. If ϵ is purely numerical, consider defining H without ϵ and stating ϵ is used only when computing logs of zero.

Limitations

- Audit is based solely on the provided 7-page parsed text; no equation numbers were available, and any formatting corruption in displayed summations may affect interpretation.

- No figures, appendices, or supplementary derivations are present in the provided text, limiting the ability to verify omitted derivation steps (e.g., precise PCA operator and covariance normalization).
- This audit does not assess numerical correctness, empirical validity, or implementation choices beyond whether the written mathematics is internally coherent.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

Method-level arithmetic and constant-value checks (counts, dimension consistency, epsilons, entropy normalization sanity check) all pass with exact or tight tolerances where computable. One methodological item ($\mathbf{ED}_{99}$) remains unverified numerically because it depends on PCA eigenvalues not provided.

Checked items

1. ✔ **C1__total__latent__vectors__per__viscosity** (Page 2, Methods 2.1 Data Description)
    - **Claim:** "The total number of latent vectors available for analysis at each viscosity is $N_x \times N_t = 101 \times 103 = 10403$."
    - **Checks:** integer__multiplication
    - **Verdict:** PASS
    - **Notes:** Exact integer product check.
2. ✔ **C2__features__dimension__decomposition** (Page 2, Methods 2.1 Data Description)
    - **Claim:** "$N_{\text{features}} = 13 \ldots$ consists of 3 mesh coordinates $(x, t, \text{and } \nu)$ followed by 10 components representing the latent space vector."
    - **Checks:** parts__sum__to__total
    - **Verdict:** PASS
    - **Notes:** Exact integer sum of parts.
3. ✔ **C3__data__array__dimension__product** (Page 2, Methods 2.1 Data Description)
    - **Claim:** "The raw data is organized as a four-dimensional array with dimensions $(N_x, N_t, N_\nu, N_{\text{features}})$, where $N_x = 101$, $N_t = 103$, $N_\nu = 25$, and $N_{\text{features}} = 13$."
    - **Checks:** integer__product__total__entries
    - **Verdict:** PASS
    - **Notes:** Derived total scalar entries computed as 3,380,975 (paper does not report an explicit total to compare).

4. ✔ **C4_coarse_grain_block_normalization** (Page 3, Methods 2.2 Spatial-Temporal Coarse-Graining Procedure (equation defining $L^{(s+1)}$))

   - **Claim:** Coarse-graining uses non-overlapping blocks of size $b_x \times b_t$ with $b_x = 2$ and $b_t = 2$, and averages with prefactor $1/(b_x b_t)$.
   - **Checks:** constant_computation
   - **Verdict:** PASS
   - **Notes:** Computed $1/(b_x \cdot b_t)$ and compared to 0.25.

5. ✔ **C5_grid_size_update_rule_from_initial** (Page 3, Methods 2.2 Spatial-Temporal Coarse-Graining Procedure)

   - **Claim:** Grid updates as $N_x^{(s+1)} = \mathrm{floor}(N_x^{(s)}/b_x)$ and $N_t^{(s+1)} = \mathrm{floor}(N_t^{(s)}/b_t)$ with initial $N_x = 101$, $N_t = 103$, $b_x = 2$, $b_t = 2$.
   - **Checks:** iterated_floor_reduction
   - **Verdict:** PASS
   - **Notes:** Iterated floor reductions from $(101, 103)$ by factors $(2, 2)$ until $(1, 1)$; verified $N = N_x \cdot N_t$ is monotone non-increasing across scales.

6. ✔ **C6_pca_components_count_matches_latent_dim** (Page 3, Methods 2.3 Latent Space Analysis at Each Scale)

   - **Claim:** PCA output includes eigenvalues and eigenvectors for $k = 1, \ldots, 10$ corresponding to the 10-dimensional latent space.
   - **Checks:** dimension_consistency
   - **Verdict:** PASS
   - **Notes:** Checked PCA component count matches latent dimension.

7. ✔ **C7_standardization_epsilon_value** (Page 3, Methods 2.3 Latent Space Analysis at Each Scale)

   - **Claim:** "A small epsilon (10^{-9}) is added to the denominator to prevent division by zero."
   - **Checks:** constant_value_check
   - **Verdict:** PASS
   - **Notes:** Checked constant equals 1×10^{-9}.

8. ✔ **C8_entropy_epsilon_value** (Page 4, Methods 2.4.3 Normalized Shannon Entropy)

   - **Claim:** Entropy computed with $\log(p_k + 10^{-9})$ for numerical stability.
   - **Checks:** constant_value_check
   - **Verdict:** PASS
   - **Notes:** Checked constant equals 1×10^{-9}; matches the standardization epsilon.

9. ✔ **C9_entropy_normalization_max** (Page 4, Methods 2.4.3 Normalized Shannon Entropy)
   - **Claim:** “The maximum possible entropy for a 10-dimensional space is $\log(10)\ldots$ We normalize... $H_{\text{normalized}} = H/\log(10)$.”
   - **Checks:** formula_sanity_check
   - **Verdict:** PASS
   - **Notes:** Uniform $p_k = 0.1$ sanity check yields $H/\log(10) = 0.999999995657055$ using epsilon=1×10^{-9} (within tolerance).
10. ⚠ **C10_effective_dimensionality_threshold_value** (Page 4, Methods 2.4.2 Effective Dimensionality (ED_{99}))
    - **Claim:** ED_{99} defined as smallest K such that cumulative explained variance $C_K \geq 0.99$.
    - **Checks:** threshold_consistency
    - **Verdict:** UNCERTAIN
    - **Notes:** Cannot compute ED *{99}orverifyC*\approx 1$ without an eigenvalue/explained-variance vector.
11. ✔ **C11_stop_threshold_min_points_for_pca** (Page 3, Methods 2.2 Spatial-Temporal Coarse-Graining Procedure)
    - **Claim:** Iteration may stop when total number of latent vectors $N_x^{(s)} \times N_t^{(s)}$ falls below a threshold “(e.g., $2 \times 10 = 20$ points) required for robust PCA.”
    - **Checks:** arithmetic_example_check
    - **Verdict:** PASS
    - **Notes:** Exact integer product check.

Limitations

- Only parsed text was provided; no tables/figures with numeric results beyond method constants, so most result claims cannot be numerically audited.
- No access to the underlying dataset or model outputs (latent vectors, eigenvalues, ED_{99}, entropy), so any checks requiring data values are listed as unverified.
- Figure placeholders contain no numeric content; plot-based value extraction is disallowed by scope.