

Skeptical review: Robust Parameter Estimation for Damped Harmonic Oscillators via Full-Trajectory Maximum Likelihood Estimation

Summary

The manuscript presents a parameter-estimation framework for underdamped oscillatory time-series by fitting the full analytic damped-sinusoid trajectory $x(t) = Ae^{-\gamma t} \cos(\omega t + \phi)$ via bounded non-linear least squares, interpreted as MLE under additive i.i.d. Gaussian noise (Sec. 1, Sec. 2.2). The implementation uses Trust Region Reflective optimization with physically motivated initialization (FFT/PSD peak for ω_0 , envelope-based estimates for A_0, γ_0) (Sec. 2.2) and evaluates performance on 20 synthetic trajectories with additive Gaussian noise (Sec. 2.1, Sec. 3). Results show small relative errors for ω and generally low errors for γ , with qualitative fits and several diagnostic plots (Sec. 3.1–3.2, Fig. 1–2, Table 1). The approach is sensible and clearly presented, but the current submission reads primarily as a proof-of-concept/implementation of a standard technique; key simulation and algorithmic details are under-specified, evaluation is limited ($N = 20$, no baselines, no uncertainty quantification), the SNR metric is potentially circular, and terminology/parameterization (natural vs damped frequency; damping ratio) needs tightening to support the broader robustness and applicability claims (Sec. 2–4).

Strengths

- Clear motivation for full-trajectory fitting as an alternative to local feature-based estimators, with a coherent MLE framing under additive Gaussian noise (Sec. 1, Sec. 2.2).
- Conceptually simple and practically useful method: bounded non-linear least squares (TRF) on an explicit analytical model (Eq. (1)), with spectral initialization and physically plausible parameter constraints (Sec. 2.2).
- Qualitative reconstructions indicate high-fidelity fits on representative noisy traces (Sec. 3.1, Fig. 1).
- Quantitative evaluation includes parity plots and analyses versus SNR and damping ratio, which help interpret when γ estimation degrades (Sec. 3.2, Fig. 2, Table 1).
- Generally clear writing and organization from problem statement through method and results (Sec. 1–4).

Major issues

1. **Positioning/novelty is currently unclear: full-trajectory non-linear least squares fitting of a damped sinusoid and its equivalence to Gaussian-noise MLE are standard in signal processing/system identification, yet the manuscript frames the work as a new “framework” without clearly identifying what is novel beyond a straightforward application (Sec. 1, Sec. 4).** The in-

roduction also lacks concrete related-work grounding, making it hard to assess contribution relative to established estimators (log decrement, Prony/matrix pencil, subspace/AR methods, Bayesian approaches, CRLB results).

Recommendation: Strengthen Sec. 1 with a compact related-work subsection (e.g., Sec. 1.1) and citations to standard damped-sinusoid estimation methods. Then explicitly state the paper’s contribution: e.g., (i) a reproducible implementation recipe with bounded TRF + a specific initialization pipeline, (ii) a benchmark/validation study under controlled conditions, (iii) (if added) uncertainty quantification, robustness experiments, or comparisons. If the intent is didactic/benchmarking, reframe claims accordingly in Abstract/Sec. 1/Sec. 4.

2. **Synthetic data generation is under-specified and the evaluation set is small ($N = 20$), limiting interpretability and generality of the accuracy/robustness claims (Sec. 2.1, Sec. 3.2).** Key missing details include distributions/ranges of $(A, \gamma, \omega, \phi)$, sampling rate/ Δt , duration and number of samples, whether sampling is uniform, whether mean removal/detrending is applied, and how noise variance is chosen across oscillators.

Recommendation: Expand Sec. 2.1 with complete, numeric simulation settings: parameter ranges/distributions (or explicit per-oscillator table in appendix), sampling frequency, duration, sample count, and noise model/variance selection. Increase trials substantially (e.g., hundreds/thousands) to support claims with distributions rather than anecdotes, and report aggregate statistics (median/IQR or mean/SD, min/max) for ω and γ errors (Sec. 3.2.1). Provide code/pseudocode sufficient for reproduction.

3. **No quantitative baseline comparisons are provided despite repeated motivation against “traditional” local feature-based estimators (Sec. 1, Sec. 3.2, Sec. 4).** Without baselines, it is impossible to determine whether full-trajectory TRF fitting materially improves accuracy/robustness or merely matches standard practice, and at what computational cost.

Recommendation: In Sec. 3, add at least two baseline estimators under identical simulation conditions: (i) FFT/PSD peak (frequency) + logarithmic decrement from peaks/envelope (damping), and (ii) a Prony/matrix-pencil or simple AR/SSA-based method (or another common system-ID approach). Report relative/absolute error vs (true) SNR and damping ratio and include runtime comparisons (median fit time per trace). Summarize in an additional table/figure.

4. **SNR definition is a post-fit statistic derived from the same fit used to compute parameter errors ($\text{SNR} = \text{Var}(\hat{x})/\text{Var}(r)$, Sec. 2.3, Eq. (3)), creating potential circularity: the optimizer can trade parameter bias against residual reduction, and the “SNR” will generally increase as fit quality increases (including possible overfitting).** As used in Sec. 3.2.2, this risks overstating explanatory power of SNR–error relationships.

Recommendation: For synthetic experiments, compute and plot error versus *true/generative* SNR defined from known clean signal and injected noise variance (or signal power/noise power prior to fitting). Keep Eq. (3) if desired, but rename it (e.g., “fit-based SNR proxy” or “variance-explained ratio”) and explicitly discuss limitations. Also report injected σ^2 vs estimated residual variance $\hat{\sigma}^2$ to validate the Gaussian-noise assumption (Sec. 3.2).

5. **Model/parameter interpretation is inconsistent: the manuscript refers to ω as “natural frequency/angular frequency” while fitting $\cos(\omega t + \phi)$, which corresponds to the *damped* oscillation frequency in the standard second-order ODE solution.** The damping ratio definition $\zeta = \gamma/\omega$ used in analysis is non-standard relative to classical $\zeta = \beta/\omega_0$ and becomes ambiguous if ω is damped (Sec. 1, Sec. 2.1–2.2, Sec. 3.2.2, Fig. 2 caption).

Recommendation: In Sec. 2.1/Sec. 2.2, explicitly define whether ω is the damped frequency ω_d or an undamped natural frequency ω_0 . If the latter, re-parameterize using the standard ODE form (e.g., $x(t) = Ae^{-\beta t} \cos(\sqrt{\omega_0^2 - \beta^2} t + \phi)$) or clearly state the mapping and adjust interpretation accordingly. Either adopt standard ζ or clearly label the current ratio as a heuristic and justify it; ensure consistency across text and figure labels (rad/s vs Hz).

6. **Key methodological details are insufficient for full reproducibility and for evaluating robustness of optimization (Sec. 2.2).** In particular: (i) envelope computation for A_0, γ_0 is vague; (ii) PSD/FFT settings for ω_0 are ambiguous (windowing, zero-padding, peak interpolation); (iii) TRF configuration (tolerances, max iterations), scaling/normalization, and convergence/failure handling are not described; (iv) treatment of ϕ (bounds/wrapping) is unclear; (v) many real signals have a DC offset not included in Eq. (1).

Recommendation: Expand Sec. 2.2 with an explicit algorithm description (or pseudocode/algorithm box): PSD computation details and peak selection; envelope method (Hilbert magnitude vs peak-picking; smoothing; log-linear regression) and bias considerations; TRF solver settings and stopping criteria; parameter vector ordering and bounds (including $\phi \in [-\pi, \pi]$ or equivalent). State whether data are detrended/mean-removed; consider adding an offset term c to the model or explicitly justify zero-mean assumption in Sec. 2.1.

7. **Uncertainty quantification is missing, despite the MLE framing (Sec. 2.2, Sec. 3, Fig. 1–2).** Reporting only point estimates and relative errors makes it difficult to judge reliability, compare regimes, or support claims about robustness—especially in low-SNR/high-damping conditions where identifiability degrades.

Recommendation: Add parameter uncertainty estimates from the Jacobian/Hessian at the optimum (Gauss–Newton covariance) and report standard errors or 95% CIs for ω, γ (and optionally A, ϕ). In simulation, perform a basic coverage check (e.g., do 95%

intervals contain the truth $\sim 95\%$ of the time across trials?). Consider adding uncertainty bands to representative fits in Fig. 1 and error bars/intervals in Fig. 2.

8. **Claims about robustness/efficiency/applicability are not fully substantiated.** The paper does not quantify runtime/scaling (despite “computationally efficient” language), and it does not probe realistic failure modes: colored/non-Gaussian noise, outliers, short observation windows (few cycles), irregular sampling/missing data, strong damping, or model mismatch (Sec. 3.2.2, Sec. 4). Identifiability issues among A, γ, ϕ (and possible offset) are not discussed.

Recommendation: Add: (i) a runtime/iteration-count report (median and range) including hardware/software, and scaling with sample count; (ii) at least one robustness experiment (e.g., colored noise, impulsive outliers with robust loss, varying window length/number of cycles, varying sampling rate) to map breakdown points; and (iii) a limitations paragraph in Sec. 4 explicitly discussing noise-model dependence, identifiability, and expected real-data complications. If robustification is not added, temper wording in Sec. 4 accordingly.

9. **Internal consistency issues between narrative claims and Table 1 examples weaken credibility: the manuscript states ω relative errors are “consistently below 0.5%,” but Table 1 includes an ω relative error of 0.0066 (0.66) for oscillator 17; similarly, a claim about γ error $< 1\%$ at $\text{SNR} > 500$ is contradicted by oscillator 1 ($\text{SNR}=630.2$, γ relative error=0.0106) (Sec. 3.2.1, Table 1, Sec. 4).**

Recommendation: Audit all thresholded performance statements in Sec. 3.2 and Sec. 4 and align them with the full dataset maxima/quantiles. If Table 1 is illustrative, either choose examples consistent with the stated thresholds or explicitly state that examples include outliers; ideally provide full-dataset summary statistics (and/or append full table) to support the claims.

10. **Figures 1–2 and Table 1 lack key context and uncertainty information needed to interpret results, and some labeling/definitions are incomplete (Sec. 3, Fig. 1–2, Table 1).** Examples: “relative error” is not always clearly defined as fraction vs percent; axes/units (rad/s, s^{-1}) are incomplete; sample size is not stated on plots; log scaling is not explicitly indicated; and fit uncertainty is not shown.

Recommendation: Revise Fig. 1–2 captions and axes to include: parameter units, definition of relative error (and whether values are fractions or %), sample size (N), and whether axes are log-scaled. Add panel labels (a–d), annotate representative traces with true/fitted parameters and (true) SNR, and include uncertainty (CIs/error bars or bands) where feasible. Ensure print legibility (font sizes, line widths) and accessibility (colorblind-safe palettes, not relying on color alone).

Minor issues

1. Notation overload/inconsistency: $x(t)$ denotes the model in Eq. (1) but later appears to denote observed data in residual definitions (Sec. 2.3). This can confuse the likelihood/MLE interpretation.

Recommendation: Introduce distinct symbols: e.g., observations $y(t_i)$, model $x(t_i; \theta)$, fitted $\hat{x}(t_i) = x(t_i; \hat{\theta})$, residuals $r_i = y_i - \hat{x}_i$. Rewrite Eq. (3) accordingly.

2. MLE equivalence is asserted but assumptions are not explicitly stated/derived: i.i.d. Gaussian noise, constant variance, independence across time points (Sec. 2.2).

Recommendation: Add the observation model $y_i = x(t_i; \theta) + \epsilon_i$, $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$ i.i.d., write the log-likelihood, and show it reduces to least squares (up to constants). State whether σ^2 is assumed known or estimated from residuals.

3. Bound constraints and parameter redundancy: constraining $A \geq 0$ is fine, but with free ϕ , negative A is redundant; ϕ bounds/wrapping are not specified (Sec. 2.2).

Recommendation: State bounds explicitly (including ϕ in $[-\pi, \pi]$ or equivalent). Briefly justify $A \geq 0$ and discuss whether alternative parameterizations (e.g., linear-in-parameters form using sine/cosine coefficients for fixed ω, γ) could help stability.

4. Phase initialization $\phi_0 = 0$ may reduce convergence reliability at low SNR or when the time origin is arbitrary (Sec. 2.2).

Recommendation: Initialize ϕ via projection onto $\cos(\omega_0 t)$ and $\sin(\omega_0 t)$ (or via analytic signal phase) after estimating ω_0 , and report whether this improves convergence/iterations.

5. Potential missing offset term: many experimental signals have DC offset/drift, but Eq. (1) has no c term and preprocessing is not specified (Sec. 2.1–2.2, Sec. 4).

Recommendation: Either (i) include c in the fitted model and bound it reasonably, or (ii) state explicit detrending/mean-removal steps and confirm synthetic data are zero-mean.

6. Table 1 is presented as a representative subset without stating selection criteria (Sec. 3.2.1).

Recommendation: State whether rows were chosen randomly, by SNR quantiles, by extremes, or as illustrative cases. If space allows, move the full table to an appendix.

7. Language occasionally overstates optimizer guarantees (e.g., “ensure robust convergence” / implied global optimum) (Sec. 2.2, Sec. 4).

Recommendation: Soften to “improves convergence reliability” and, if possible, report empirical failure rate (non-convergence, boundary solutions) over many trials.

Very minor issues

1. Formatting/structure issues: stray heading marker (“###”) before Sec. 2.2; inconsistent subsection heading styles; figure/table captions sometimes appear disconnected or include placeholder file names (Sec. 2.2, Sec. 3; Fig. 1–2; Table 1).

Recommendation: Clean LaTeX/markdown structure, ensure figure/table environments are correctly linked to captions, remove placeholder file names, and verify cross-references resolve to the correct items.

2. Minor style consistency: hyphenation (“time series” vs “time-series”), terminology shifts (natural frequency vs frequency), occasional spacing/punctuation issues, and equations could be formatted more readably (Sec. 1–4).

Recommendation: Perform a consistency pass: standardize terms and hyphenation, remove double spaces, and format Eq. (1)–(3) as display equations with consistent numbering and spacing.

3. Figure accessibility/readability: reliance on color alone, potentially small fonts/lines for print, identity line visibility in parity plots, and unclear tick formatting on (possibly) log axes (Fig. 1–2).

Recommendation: Use colorblind-safe palettes plus marker/linestyle redundancy, increase font/line sizes, thicken identity lines (optionally add $\pm x\%$ bands), add light gridlines on log axes, and ensure tick labels clearly indicate scaling.

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: light

The paper’s mathematics consists primarily of a parametric signal model for a decaying sinusoid (Eq. (1)), a relative-error definition (Eq. (2)), and an SNR definition based on variance ratios (Eq. (3)), plus related definitions (residuals, damping ratio) and qualitative statements about least-squares/MLE equivalence. No multi-step derivations are shown.

Checked items

1. ✓ **Damped oscillator trajectory model** (Eq. (1), Sec. 2.2, p.3)
 - **Claim:** The displacement trajectory is modeled as $x(t) = Ae^{-\gamma t} \cos(\omega t + \phi)$ with parameters A, γ, ω, ϕ .
 - **Checks:** symbol/definition consistency, dimensional/units sanity
 - **Verdict:** PASS; confidence: high; impact: critical

- **Assumptions/inputs:** System is underdamped so an exponentially decaying sinusoid is an appropriate trajectory form, A , γ , ω are nonnegative/positive as later constrained
 - **Notes:** Exponent γt must be dimensionless, so γ has units 1/time; ωt dimensionless so ω has units 1/time (radians treated as dimensionless). This is consistent with the model form.
2. ✓ **Free-parameter identification for Eq. (1)** (Sec. 2.2, immediately after Eq. (1), p.3)
- **Claim:** The model has four free parameters: A , γ , ω , ϕ .
 - **Checks:** notation consistency
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** No additional offset term is included
 - **Notes:** Consistent with Eq. (1) as written.
3. △ **MLE equivalence to least squares (Gaussian noise)** (Sec. 2.2, p.3)
- **Claim:** Under additive Gaussian noise, fitting by non-linear least squares is equivalent to Maximum Likelihood Estimation.
 - **Checks:** derivation logic completeness
 - **Verdict:** UNCERTAIN; confidence: medium; impact: moderate
 - **Assumptions/inputs:** Observed data are the model plus additive Gaussian noise, Typically requires independent noise with common variance across time points
 - **Notes:** The paper does not write the observation model or likelihood, so the equivalence cannot be verified from the document alone. The needed missing piece is an explicit probabilistic statement (e.g., $y_i = x(t_i; \theta) + \epsilon_i$) and the resulting likelihood/log-likelihood expression.
4. ✓ **Relative error definition** (Eq. (2), Sec. 2.3, p.3)
- **Claim:** Relative Error = $|p_{\text{fit}} - p_{\text{true}}|/p_{\text{true}}$.
 - **Checks:** algebraic form, well-posedness
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** $p_{\text{true}} \neq 0$ (here p is a positive physical parameter)
 - **Notes:** Dimensionless and standard. Would be undefined if $p_{\text{true}} = 0$, but the paper constrains $\gamma \geq 0$ and $\omega > 0$ and uses positive ground-truth values in context.
5. ✗ **Residual definition vs model/data notation** (Residual definition in Sec. 2.3, p.3)
- **Claim:** Residuals are $r(t) = x(t) - \hat{x}(t)$, where $\hat{x}(t)$ is the fitted model signal.

- **Checks:** symbol/definition consistency
- **Verdict:** FAIL; confidence: high; impact: moderate
- **Assumptions/inputs:** $x(t)$ in this expression denotes observed data, not the analytical model
- **Notes:** Earlier, $x(t)$ is explicitly the analytical model (Eq. (1)). In Sec. 2.3, $x(t)$ must represent the observed noisy data to make $r(t)$ meaningful. This is a notational inconsistency that should be resolved by using different symbols for observed data and the model.

6. ✓ **SNR definition via variance ratio** (Eq. (3), Sec. 2.3, p.3)

- **Claim:** $\text{SNR} = \text{Var}(\hat{x}(t))/\text{Var}(r(t))$, with $r(t) = x(t) - \hat{x}(t)$.
- **Checks:** dimensional/units sanity, definition consistency (conditional on residual definition)
- **Verdict:** PASS; confidence: medium; impact: minor
- **Assumptions/inputs:** Variance is computed consistently over the same time index set for \hat{x} and r
- **Notes:** As a definition, the ratio is dimensionless and internally coherent once the data/model notation for $r(t)$ is clarified. The paper does not specify whether Var is population vs sample variance, but that is not an algebraic inconsistency.

7. ✓ **Damping ratio definition** (Sec. 3.2.2, p.5)

- **Claim:** Damping ratio is defined as $\zeta = \gamma/\omega$.
- **Checks:** dimensional/units sanity, symbol consistency
- **Verdict:** PASS; confidence: medium; impact: minor
- **Assumptions/inputs:** ω is the frequency parameter used in the cosine term of Eq. (1)
- **Notes:** γ and ω both have units $1/\text{time}$, so ζ is dimensionless (treating radians as dimensionless). This is consistent internally, though the term 'damping ratio' could be clarified relative to the intended ω .

8. ✓ **Optimization bounds consistency with model** (Sec. 2.2, end, p.3)

- **Claim:** Optimization is subject to bounds $A > 0, \gamma \geq 0, \omega > 0$ to guarantee physical validity.
- **Checks:** constraint/model compatibility
- **Verdict:** PASS; confidence: high; impact: minor
- **Assumptions/inputs:** Model amplitude A is taken nonnegative (sign absorbed into phase ϕ), Nonnegative damping rate corresponds to decaying or non-growing envelopes

- **Notes:** These bounds are compatible with Eq. (1). $A > 0$ is not strictly necessary mathematically (negative A can be absorbed into ϕ), but it is consistent.

9. **⚠ Initial-guess parameter vector ordering** (Sec. 2.2, middle, p.3)

- **Claim:** Initial guess is the parameter vector $(\omega_0, \gamma_0, A_0, \phi_0)$.
- **Checks:** notation consistency
- **Verdict:** UNCERTAIN; confidence: medium; impact: minor
- **Assumptions/inputs:** The optimizer uses the same ordering internally
- **Notes:** Within the text, the natural parameter listing is $(A, \gamma, \omega, \phi)$, but the initial vector is presented in a different order. This is not an algebraic error, but it is ambiguous without an explicit definition of θ and its ordering.

Limitations

- Audit is based only on the provided 7-page text/images; no additional appendices or derivations are available to verify omitted steps (e.g., explicit likelihood).
- Figures and tables are not audited numerically; only symbol/definition consistency around them is considered.
- No governing differential equation is presented, so consistency between Eq. (1) and a specific physical ODE parameterization cannot be checked from the document alone.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

Audited internal consistency of several numeric claims and thresholds against the explicitly provided Table 1 subset values and repeated constants. Two key narrative thresholds ($\omega < 0.5\%$ and high-SNR $\Rightarrow \gamma < 1\%$) are contradicted by Table 1 entries; repeated constants (dataset size 20; 20-second interval) are consistent. Several additional claims tied to Figure 2 or full per-oscillator data could not be verified from the provided text/table subset.

Checked items

1. **⚠ C1_relerr_formula_eq2_sign** (Page 3, Eq. (2) under '2.3 Evaluation metrics')
 - **Claim:** Relative Error = $|p_{\text{fit}} - p_{\text{true}}|/p_{\text{true}}$
 - **Checks:** formula_sanity_nonnegativity
 - **Verdict:** UNCERTAIN
 - **Notes:** Non-negativity/scale invariance depends on whether p_{true} is guaranteed positive for every parameter to which the relative error is applied; provided inputs note $\gamma \geq 0$ and $\omega > 0$, but no complete parameter-domain coverage is available here.
2. **✓ C2_snr_formula_var_ratio** (Page 3, Eq. (3) under '2.3 Evaluation metrics')

- **Claim:** $\text{SNR} = \text{Var}(\hat{x}(t))/\text{Var}(r(t))$ where $r(t) = x(t) - \hat{x}(t)$
 - **Checks:** formula_internal_consistency
 - **Verdict:** PASS
 - **Notes:** As stated, the variance ratio is dimensionless and non-negative when $\text{Var}(r(t)) > 0$; no conflicting alternative (e.g., dB) definition is provided in the inputs.
3. ✘ **C3_table1_gamma_relerr_range_claim** (Page 6, Table 1; Page 5, Section 3.2.1; Page 6, Conclusions)
- **Claim:** Paper claims damping rate γ relative errors typically within 1–3% (also stated as 1% to 3%); Table 1 lists γ relative errors for subset oscillators.
 - **Checks:** range_check_against_table_subset
 - **Verdict:** FAIL
 - **Notes:** Table 1 includes γ relerr=0.0318, exceeding 0.0300 by 0.0018 (> 0.0005 tolerance). Several other subset values are far below 0.01, which does not contradict 'within 1–3%' mathematically but weakens the 'typically 1–3%' phrasing for a representative subset.
4. ✘ **C4_table1_omega_relerr_vs_0p5pct_claim** (Page 5, Section 3.2.1; Page 6, Table 1; Page 1 Abstract; Page 6 Conclusions)
- **Claim:** Paper claims ω relative errors consistently below 0.5%; Table 1 lists ω relative errors for subset oscillators.
 - **Checks:** threshold_check
 - **Verdict:** FAIL
 - **Notes:** Table 1 shows ω relerr=0.0066 for oscillator 17, which exceeds 0.005 by 0.0016.
5. ✘ **C5_subset_claim_high_snr_less_1pct_gamma** (Page 5, Section 3.2.2)
- **Claim:** For instance, oscillators with high SNR (e.g., $\text{SNR} > 500$) consistently yield γ estimates with less than 1% error.
 - **Checks:** conditional_threshold_check_against_table_subset
 - **Verdict:** FAIL
 - **Notes:** In Table 1, oscillator 1 meets $\text{SNR} > 500$ (630.2) but has γ relerr=0.0106, exceeding the < 0.01 threshold by 0.0006.
6. ✔ **C6_counts_dataset_size_mentions** (Page 1 Abstract; Page 2 Section 2.1; Page 4 Section 3.1; Page 6 Conclusions)
- **Claim:** Dataset size is stated as 20 simulated oscillators in multiple places.
 - **Checks:** repeated_constant_consistency
 - **Verdict:** PASS

- **Notes:** All provided mentions are **20**.
7. ✓ **C7_time_interval_mentions_20_seconds** (Page 2 Section 2.1; Page 4 Figure 1 caption; Page 4 Section 3.1)
- **Claim:** Signals are sampled/fit over a **20**-second interval; multiple mentions of '20-second interval/duration'.
 - **Checks:** repeated_constant_consistency
 - **Verdict:** PASS
 - **Notes:** All provided mentions are **20** seconds.

Limitations

- Only parsed text and embedded table values were available; the paper's plots (Figures 1–2) contain additional numerical information but extracting values from plot pixels is out of scope.
- No per-oscillator ground-truth parameters, fitted parameters, or full dataset outputs are provided in text, preventing recomputation of relative errors, SNR, or correlations for the full set of **20** oscillators.
- Some checks are limited to internal consistency between narrative claims and the small 'representative subset' in Table 1; subset agreement does not prove or disprove whole-dataset claims, but can still surface contradictions.
- Execution environment error prevented running automated checks: Sandbox policy violation: from-import of 'typing' is not allowed.