Skepthical review: *Quantifying the Temporal Limits of Parameter Identifiability in Damped Harmonic Oscillators*

Summary

The paper studies how the *time window of observation* affects which parameters of an underdamped harmonic oscillator are most influential/"identifiable," using the total mechanical energy signal $E(t) = \frac{1}{2}mv(t)^2 + \frac{1}{2}kx(t)^2$ (Sec. 1–2, Eq. (1)). Across 20 simulated oscillators, the authors compute Jacobian-based local sensitivities of $E(t)$ with respect to mass m and damping b (Sec. 2.2), define an "Information Horizon" T_H as the first time the damping sensitivity overtakes the mass sensitivity via $R(t) = S_b(t)/S_m(t) = 1$ (Sec. 2.3), and report that T_H occurs early (mean ≈ 0.76 s), with larger damping ratio ζ yielding earlier T_H and lower peak sensitivity (Sec. 3.1–3.3, Sec. 4). The overall idea—making explicit that parameter influence can be time-dependent and thus relevant to experiment design—is well-motivated and clearly presented. However, several core elements are currently under-specified or conceptually overstated: the dynamical/simulation setup and sensitivity computation (especially $\partial E/\partial b$) are not reproducible as written; the sensitivity measures and the horizon criterion are dimensionally inconsistent without normalization; and the manuscript's use of "identifiability/information" is not yet tied to a measurement/noise model or an inference procedure. Addressing these points (and clarifying what is assumed measured: E vs x, v) would substantially strengthen both rigor and practical relevance.

Strengths

- Well-motivated focus on *time-dependent* parameter influence/identification windows, with a clear narrative in the Introduction (Sec. 1) and a coherent end-to-end workflow (Sec. 2 → Sec. 3).
- The "Information Horizon" concept is intuitive and potentially useful as an experiment-design diagnostic (Sec. 2.3, Sec. 3.1).
- Using time-resolved sensitivities to illustrate early-transient vs late-time dominance is pedagogically effective and aligns with physical intuition for decaying oscillations (Sec. 3.1–3.2).
- Figures 1–2 convey the qualitative trends (early large sensitivity, transition from mass- to damping-dominated behavior), and the damping-ratio trend analysis is a reasonable first attempt at summarizing across systems (Sec. 3.3).
- The paper is generally readable and consistent in high-level notation for $E(t)$ and the Jacobian idea (Eqs. (1)–(4)), making the approach accessible to a broad audience.

Major issues

1. **Reproducibility gap: key dynamical model and simulation details are missing or ambiguous (Sec. 2.1), making headline quantitative claims (e.g., mean $T_H \approx 0.76\,\mathrm{s}$, range $[0.60, 0.92]\,\mathrm{s}$, regression/correlation in Sec. 3.3) non-auditable.** The manuscript does not clearly specify the governing equation (e.g., $m\ddot{x} + b\dot{x} + kx = 0$ vs forcing), the exact parameterization used to generate the 20 oscillators (ranges/distributions for m, b, k, any constraints such as fixed ω_n), initial conditions $x(0), v(0)$, solver/integration method, time step/sampling rate, and number of discrete time points used in sensitivity calculations.

   Recommendation: In Sec. 2.1, explicitly state: (a) the ODE (and whether any forcing/noise is present); (b) the initial conditions and how they are chosen; (c) whether solutions are analytic or numerical and, if numerical, the solver, Δt, and simulation duration; (d) the discrete sampling grid for $E(t)$ and sensitivities; and (e) how the 20 oscillator parameter sets are generated (ranges/distributions, any coupling/constraints, and resulting ranges of ω_n and ζ). Provide a parameter table (main text or supplement) so Sec. 3.1–3.3 statistics can be independently reproduced.

2. **Central methodological ambiguity: $\partial E/\partial b$ (and in general "Jacobian of E") is not defined/computed in a mathematically checkable way (Sec. 2.2–2.3).** As written, Eq. (1) has no explicit dependence on b, so $\partial E/\partial b \neq 0$ only through the implicit dependence $x(t;b), v(t;b)$. The paper does not state whether derivatives are computed via sensitivity ODEs, closed-form differentiation, automatic differentiation, or finite differences; nor does it discuss numerical error, step-size selection, smoothing, or stability. This affects $S_b(t)$, $R(t)$, and therefore T_H (Sec. 3.1–3.2).

   Recommendation: In Sec. 2.2, define $E(t; m, b, k) = \frac{1}{2} m v(t; m, b, k)^2 + \frac{1}{2} k x(t; m, b, k)^2$ and explicitly state that derivatives are total derivatives through the state: e.g., $\frac{\partial E}{\partial b} = mv\,\frac{\partial v}{\partial b} + kx\,\frac{\partial x}{\partial b}$. Then specify how $\frac{\partial x}{\partial b}, \frac{\partial v}{\partial b}$ (and similarly for m) are obtained: (i) forward sensitivity equations (include the augmented ODEs in-text or appendix), or (ii) finite differences (scheme, perturbation size, convergence checks), or (iii) AD (tooling + validation). Also clarify whether $\partial E/\partial m$ includes both the explicit term $\frac{1}{2}v^2$ and the implicit dependence of x, v on m.

3. **Dimension/unit inconsistency: the sensitivity norm $S(t) = |J(t)|_2$ and the ratio $R(t) = S_b(t)/S_m(t)$ compare/aggregate derivatives with different physical units (Sec. 2.2–2.3; Eq. (3)/(6), Eq. (4)/(5); Figures 1–2).** As a result, both the magnitude of $S(t)$ and the threshold $R(t) = 1$ are parameterization- and unit-dependent, undermining the quantitative meaning of "dominance" and comparability across oscillators.

   Recommendation: Replace raw derivatives with dimensionless or uncertainty-weighted quantities. Options include: (a) relative sensitivities $\tilde{S}_m = |\frac{m}{E}\frac{\partial E}{\partial m}|$, $\tilde{S}_b = |\frac{b}{E}\frac{\partial E}{\partial b}|$; (b) log-sensitivities $|\partial \ln E/\partial \ln \theta|$; or (c) error-propagation/FIM-style weighting with parameter prior uncertainties σ_m, σ_b: $\mathrm{Var}[E(t)] \approx J(t)\Sigma_\theta J(t)^\top$. Redefine T_H using equality of *comparable* (dimensionless/weighted) contributions and update Figures 1–2 axes/units accordingly.

4. **"Identifiability/information" claims are not yet supported by an explicit measurement model, noise model, or estimation procedure (Sec. 1, Sec. 2.2–2.4, Sec. 3.1, Sec. 4).** Local sensitivity magnitude is related to parameter influence but is not equivalent to structural/practical identifiability, nor to "information" in the Fisher sense. Moreover, the discussion does not address that later times may have higher *relative* damping sensitivity but lower SNR due to decay, which can reduce practical information under noise (big-picture concern raised by the current framing in Sec. 4).

   Recommendation: Either (A) reframe the paper consistently as *time-varying local sensitivity of the energy output* and soften language in Abstract/Sec. 1/Sec. 4 (avoid "fundamental limit," "operational limit," etc.), or (B) add a minimal inference layer: assume a noise model for the observable (E or x, v), compute a time-window Fisher information (or cumulative information) for m and b, and/or run a parameter-estimation experiment (fit m, b on early vs late windows) showing estimation variance/bias changes around T_H. If retaining the term "Information Horizon," justify it via an information measure (even in a simplified setting).

5. **Observability/measurement realism: it is unclear what is assumed measured and how $E(t)$ is formed when m is unknown (Sec. 1–2).** In many setups, $E(t)$ is not directly measured; it is derived from $x(t), v(t)$ and parameters (including m and k). If m is the unknown, constructing $\frac{1}{2}mv^2$ already requires m, creating a circularity unless E is treated as a model-predicted quantity rather than a direct measurement.

   Recommendation: Clarify the observation model in Sec. 2.1–2.2: is the 'output' assumed to be (i) directly measured energy (with what sensor/proxy), (ii) energy computed from measured x, v using nominal parameters, or (iii) purely model-based analysis of E as a derived signal? If the goal is system identification, consider also presenting results for more standard observables ($x(t)$, $v(t)$, or envelope/decay rate) or discuss explicitly why energy is the chosen output and what is gained/lost compared to displacement-based identification.

6. **The operational definition of T_H and the definitions of S_m, S_b are inconsistent/underspecified (Sec. 2.2 vs Sec. 3.1), affecting the existence and timing of the horizon.** The manuscript alternates between absolute-value and signed derivatives for S_m, S_b; does not specify how T_H is extracted on a discrete time grid; and does not describe handling for multiple crossings, non-crossing cases, or numerical oscillations in $R(t)$.

Recommendation: Unify the definition of S_m, S_b across Sec. 2.2–2.3 and Sec. 3.1 (preferably magnitudes if the goal is "dominance"). In Sec. 2.3, provide an explicit algorithm for T_H: e.g., "the smallest sampled time where $R(t) \geq 1$, with linear interpolation between neighboring samples," plus rules for multi-crossings (first sustained crossing; smoothing/hysteresis), and for non-crossing trajectories (report as missing and discuss). Report variability for T_H (SD/CI) in Sec. 3.1 and mention any atypical cases.

7. **Scope limitations are not sufficiently reflected in conclusions: the study uses 20 noise-free simulations and focuses on only m and b while stiffness k is present in both dynamics and energy (Sec. 2.1–2.2, Sec. 3, Sec. 4).** The exclusion of k is not justified, and robustness to parameter ranges, initial conditions, or measurement noise is not examined, yet conclusions are phrased broadly.

   Recommendation: In Sec. 2.2 and Sec. 4, explicitly justify excluding k (e.g., assumed known/calibrated; focus on inertial vs dissipative). Ideally add $\partial E/\partial k$ and discuss whether multiple crossovers/horizons appear when k is included. Add at least one robustness check: more oscillators ($n \gg 20$), alternative parameter distributions (including very low and near-critical damping), alternative initial conditions/amplitudes, and a simple measurement-noise experiment to test whether T_H remains meaningful when amplitude decays.

8. **Positioning within existing literature is thin, making novelty and interpretation of "Information Horizon" unclear (Sec. 1–4).** There is no dedicated Related Work section tying the approach to established sensitivity analysis for ODEs, structural/practical identifiability, observability windows, or Fisher-information-based time-window design.

   Recommendation: Add a short Related Work subsection (end of Sec. 1 or as Sec. 2.5) covering: (i) local/global sensitivity analysis in dynamical systems, (ii) identifiability/observability for oscillatory second-order systems, and (iii) time-window selection/optimal experimental design via Fisher information. In Sec. 1 and Sec. 4, position T_H explicitly as a heuristic crossover metric (unless upgraded to an information-based measure per above).

Minor issues

1. Definition of damping ratio ζ is not stated, and its interaction with ω_n is unclear (Sec. 2.1, Sec. 2.4, Sec. 3.3). This complicates interpretation of the reported linear relationship between T_H (or $S_{\max}$) and ζ.

   Recommendation: In Sec. 2.1, define ζ explicitly (e.g., $\zeta = b/(2\sqrt{km})$) and state whether $\omega_n = \sqrt{k/m}$ is held fixed or varies. In Sec. 3.3, briefly discuss whether the trend persists when controlling for ω_n or using a different parameterization (e.g., fixed ω_n, varying ζ).

2. Statistical reporting for regressions/correlations is incomplete given $n = 20$ (Sec. 2.4, Sec. 3.3). The paper reports slope, R^2, and Pearson ρ but not uncertainty or assumption checks.

   Recommendation: Add confidence intervals/standard errors and p-values for slopes and correlations (Sec. 3.3), and include the scatter plots used for the reported fits. Note explicitly that results are indicative due to small n, or increase n to stabilize estimates.

3. Figures: Figure 1 is visually cluttered and does not directly substantiate the T_H claim (crossings are not annotated); Figure 2 heatmap lacks ordering/annotations that would reveal cohort structure (Sec. 3.1–3.2).

   Recommendation: In Figure 1, add vertical markers for T_H per trajectory (or show median/quantiles with shaded bands) and/or include an inset histogram of T_H. Consider an early-time zoom (e.g., 0–2 s). In Figure 2, order oscillators by ζ or T_H and overlay T_H markers to make patterns interpretable.

4. Key reported numeric claims are not easily auditable (Sec. 3.1–3.3): the list of per-oscillator T_H values and the per-oscillator pairs used for regressions/correlations are not provided.

   Recommendation: Provide a table or supplement with per-oscillator parameters (m, b, k, ζ, ω_n), computed T_H, $S_{\max}$, and any regression inputs so readers can reproduce Sec. 3.1–3.3.

5. Terminology: "energy manifold" is used though $E(t)$ is treated as a scalar time series (Sec. 1, Sec. 2.2, Sec. 3.2, Sec. 4).

   Recommendation: Replace with "energy trajectory/signal" or define precisely what manifold is meant (e.g., surface over (t, θ)).

6. Notation consistency: $S(t)$ is defined in Sec. 2.2 and reintroduced in Sec. 3.2; S_m, S_b switch between absolute and signed forms (Sec. 2.2 vs Sec. 3.1).

   Recommendation: Cross-reference earlier definitions when restating (e.g., "recalling Eq. (3)…"), and enforce one convention for signed vs magnitude sensitivities throughout equations, text, and captions.

Very minor issues

1. Presentation/typography inconsistencies (headings, quotation marks for "Information Horizon," unit spacing such as "1s" vs "1 s," and R^2 formatting) across Sec. 2–4 and figure captions.

   Recommendation: Standardize section heading styles, quotation mark style, and SI unit formatting (e.g., "$t < 1\,\mathrm{s}$"). Use consistent mathematical typesetting for T_H, ζ, and R^2 in text, tables, and captions.

2. Norm notation $|\cdot|_2$ is used without explicitly stating it is the Euclidean norm (Sec. 2.2).

   Recommendation: Add a brief note after Eq. (3) that $|\cdot|_2$ denotes the Euclidean (L2) norm.

3. Figure labeling polish: legends/units are not always explicit (e.g., Figure 1 y-axis units/dimensionless status; Figure 2 colorbar tick labeling).

   Recommendation: Ensure all axes and colorbars clearly state units or explicitly state nondimensionalization; add in-plot legends and meaningful colorbar tick values.

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: substantial

The paper's analytic core is a Jacobian-based sensitivity analysis of oscillator energy with respect to parameters (mass and damping), and a derived crossover time ("Information Horizon") defined when the damping sensitivity overtakes mass sensitivity. The main mathematical concerns are (i) missing definitions/derivations for parameter derivatives through the implicit dependence of $x(t)$, $v(t)$ on parameters, especially for damping b, and (ii) dimensional inconsistency in taking Euclidean norms and ratios of derivatives taken with respect to parameters of different units.

Checked items

1. ✔ **Total mechanical energy definition** (Eq. (1), Sec. 2.1, p.2)

   - **Claim:** Total mechanical energy is $E(t) = 1/2mv(t)^2 + 1/2kx(t)^2$.
   - **Checks:** symbol/notation consistency, dimensional/units sanity
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** $x(t)$ is displacement, $v(t) = dx/dt$ is velocity, m is mass, k is stiffness
   - **Notes:** Standard kinetic + potential energy expression; units are consistent if v is velocity and x is displacement.

2. ⚠ **Jacobian definition with respect to parameters** (Eq. (2), Sec. 2.2, p.3)

   - **Claim:** Define $J(t) = [\partial E/\partial m, \partial E/\partial b]^T$.
   - **Checks:** definition consistency, implicit-dependence check
   - **Verdict:** UNCERTAIN; confidence: medium; impact: critical
   - **Assumptions/inputs:** E depends on parameters through $x(t)$, $v(t)$ and possibly explicitly, Partial derivatives are evaluated at each time t

- **Notes:** Given Eq. (1), E has no explicit b-dependence; $\partial E/\partial b$ is only nonzero if $x(t)$,$v(t)$ depend on b and the chain rule is used. The paper does not specify whether derivatives treat x, v as fixed or include state sensitivities, nor does it provide the needed sensitivity equations/closed-form expressions.

3. ✖ **Sensitivity index as Euclidean norm** (Eq. (3), Sec. 2.2, p.3)
    - **Claim:** Overall sensitivity is $S(t) = |J(t)|_2$.
    - **Checks:** dimensional/units consistency, definition adequacy
    - **Verdict:** FAIL; confidence: high; impact: critical
    - **Assumptions/inputs:** $J(t)$ is treated as a vector in a metric space with standard Euclidean metric
    - **Notes:** The components $\partial E/\partial m$ and $\partial E/\partial b$ have different physical units (energy per mass vs energy per damping). A Euclidean norm implicitly assumes commensurate units/scales; without normalization/weighting, $S(t)$ is not dimensionally meaningful and depends on parameter units/parameterization.

4. ✔ **Partial sensitivities as magnitudes** (Sec. 2.2, p.3)
    - **Claim:** Define $S_m(t) = |\partial E/\partial m|$ and $S_b(t) = |\partial E/\partial b|$.
    - **Checks:** notation consistency
    - **Verdict:** PASS; confidence: high; impact: minor
    - **Assumptions/inputs:** Absolute value is intended to measure magnitude of sensitivity
    - **Notes:** Definition is internally clear, but later sections drop absolute values (see separate item).

5. ✖ **Sensitivity ratio definition** (Eq. (4), Sec. 2.3, p.3)
    - **Claim:** Define $R(t) = S_b(t)/S_m(t)$.
    - **Checks:** dimensional/units consistency, well-posedness (scale invariance)
    - **Verdict:** FAIL; confidence: high; impact: critical
    - **Assumptions/inputs:** S_m, S_b are comparable quantities
    - **Notes:** $R(t)$ is not dimensionless because S_b and S_m are derivatives with respect to parameters of different units; thus the ratio depends on the chosen units/parameter scaling. The criterion $R(t) = 1$ is arbitrary unless a normalization (e.g., relative or uncertainty-weighted sensitivities) is introduced.

6. ✖ **Information Horizon definition via $R(t) = 1$** (Sec. 2.3, p.3)
    - **Claim:** Define T_H such that $R(T_H) = 1$ (crossover where damping sensitivity exceeds mass sensitivity).
    - **Checks:** definition adequacy, units/scale invariance, edge-case sanity
    - **Verdict:** FAIL; confidence: high; impact: critical

- **Assumptions/inputs:** $R(t)$ is dimensionless and scale-invariant, $R(t)$ is well-defined ($S_m(t) \neq 0$ at crossover), $R(t)$ crosses 1 (monotonicity not required but crossing must exist)
- **Notes:** Because $R(t)$ is not dimensionless, the horizon time T_H is not invariant to unit changes (e.g., rescaling b or m units) and is therefore not a mathematically well-defined, physical crossover without normalization. Also, existence/uniqueness conditions for T_H are not discussed (e.g., if $S_m(t) = 0$ at some t or multiple crossings occur).

7. ✔ **Reuse of ratio in Results** (Eq. (5), Sec. 3.1, p.4)
   - **Claim:** Restates $R(t) = S_b(t)/S_m(t)$ for the horizon discussion.
   - **Checks:** definition consistency
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** Same $R(t)$ as in Methods
   - **Notes:** Consistent restatement, though it inherits the dimensional inconsistency noted for Eq. (4).

8. ✔ **Expanded norm formula** (Eq. (6), Sec. 3.2, p.4)
   - **Claim:** $S(t) = \sqrt{S_m(t)^2 + S_b(t)^2}$.
   - **Checks:** algebra between definitions, notation consistency
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** $S(t) = |J|_2$ and S_m, S_b are the two components (or magnitudes) used in the norm
   - **Notes:** Algebra matches the 2-norm for a 2D vector. However, the dimensional inconsistency of mixing units remains (see Eq. (3) item).

9. ⚠ **Inconsistency of absolute values in Results** (Sec. 3.1, p.4)
   - **Claim:** States $S_m(t) = \partial E/\partial m$ and $S_b(t) = \partial E/\partial b$ (without absolute values) after earlier defining magnitudes.
   - **Checks:** definition consistency
   - **Verdict:** UNCERTAIN; confidence: high; impact: moderate
   - **Assumptions/inputs:** Either signed or magnitude convention is intended
   - **Notes:** If S_m, S_b are meant to be magnitudes, dropping $|\cdot|$ is a notation error; if signed derivatives are intended, then $R(t)$ could be negative and the horizon criterion needs revision. The manuscript does not clarify which is used in computations/figures.

10. ✔ **Peak sensitivity definition** (Sec. 2.4, p.3)
    - **Claim:** Define $S_{\max} = \max(S(t))$.
    - **Checks:** definition consistency
    - **Verdict:** PASS; confidence: high; impact: minor

- **Assumptions/inputs:** $S(t)$ is defined for all sampled times and bounded
    - **Notes:** Well-defined as a functional of $S(t)$ over a finite time window.

11. ⚠ **Damping ratio symbol usage** (Sec. 2.1 and Sec. 3.3, pp.2 and 5-6)
    - **Claim:** Uses ζ as a damping ratio to characterize the damping regime and relate it statistically to T_H and $S_{\max}$.
    - **Checks:** missing definition check, symbol consistency
    - **Verdict:** UNCERTAIN; confidence: high; impact: moderate
    - **Assumptions/inputs:** ζ is computed from (m, b, k) in a consistent way across oscillators
    - **Notes:** ζ is never defined mathematically in the text provided, preventing verification of any analytic linkages or constraints (e.g., underdamped condition).

Limitations

- The provided PDF text does not include the governing differential equation for the damped oscillator or any explicit solution $x(t)$, $v(t)$; without this, $\partial x/\partial m$, $\partial x/\partial b$, $\partial v/\partial m$, $\partial v/\partial b$ (and thus $\partial E/\partial m$, $\partial E/\partial b$ under implicit dependence) cannot be analytically verified.
- Figures are referenced but not analytically checkable from text alone; no explicit formulas for $S_m(t)$, $S_b(t)$ trajectories are given.
- No parameter-uncertainty model (e.g., σ_m, σ_b) is specified, which is necessary to make cross-parameter comparisons dimensionally meaningful.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

No executable numeric checks were completed (execution error). Several internal-consistency targets were identified (ranges/thresholds and cross-referenced constants), but regression/correlation and distributional claims remain unverified without underlying data.

Checked items

1. ⚠ **C1** (Page 4 (Results §3.1): 'mean time of 0.76 s ... between 0.60 s and 0.92 s')
    - **Claim:** Across the population of oscillators, the Information Horizon occurred at a mean time of 0.76 s, with all instances occurring between 0.60 s and 0.92 s.
    - **Checks:** range_contains_mean
    - **Verdict:** UNCERTAIN
    - **Notes:** Planned check: verify $\mathbf{min}\, T_H \le \mathbf{mean}_{T_H} \le \mathbf{max}\, T_H$ using the stated values (0.60, 0.76, 0.92). Not executed due to runner error.

2. ⚠ **C2** (Page 6 (Table 1) vs Page 4 (§3.1) vs Page 7 (Conclusions): Mean T_H)

   - **Claim:** Mean Information Horizon (T_H) is reported as 0.76 s in multiple sections (Results/Table 1/Conclusions).
   - **Checks:** cross_reference_constant_match
   - **Verdict:** UNCERTAIN
   - **Notes:** Planned check: confirm all reported mean T_H values match within rounding tolerance. Not executed due to runner error.

3. ⚠ **C3** (Page 2 (§2.1) vs Page 4/6: simulation window)

   - **Claim:** The paper states a population of 20 oscillators simulated over a 20-second window/interval, repeated across sections/captions.
   - **Checks:** cross_reference_integer_match
   - **Verdict:** UNCERTAIN
   - **Notes:** Planned check: equality of oscillator count and simulation duration across references. Not executed due to runner error.

4. ⚠ **C4** (Page 4 (Eq. 6) vs Page 3 (§2.2): definition of $S(t)$)

   - **Claim:** $S(t)$ is defined as the Euclidean norm of the Jacobian vector; later written explicitly as $S(t) = \sqrt{S_m(t)^2 + S_b(t)^2}$.
   - **Checks:** algebraic_equivalence
   - **Verdict:** UNCERTAIN
   - **Notes:** Planned symbolic consistency check ($|J|_2$ vs $\sqrt{S_m^2 + S_b^2}$ given $S_m = |\partial E/\partial m|$ and $S_b = |\partial E/\partial b|$). Not executed due to runner error.

5. ⚠ **C5** (Page 3 (§2.3) vs Page 4 (§3.1): Information Horizon condition)

   - **Claim:** Information Horizon T_H is defined where $R(T_H) = 1$ with $R(t) = S_b(t)/S_m(t)$.
   - **Checks:** definition_consistency
   - **Verdict:** UNCERTAIN
   - **Notes:** Planned definition match check across sections ($R(t) = S_b/S_m$ and horizon condition $R(T_H) = 1$). Not executed due to runner error.

6. ⚠ **C6** (Page 5 (§3.3) vs Page 6 (Table 1): regression/correlation values)

   - **Claim:** The slope, R^2, and correlation are stated in text and repeated in Table 1; they should match numerically.
   - **Checks:** cross_reference_constant_match
   - **Verdict:** UNCERTAIN
   - **Notes:** Planned check: compare text vs Table 1 for slope -0.8347, R^2 0.3326, ρ -0.5221 within tolerance. Not executed due to runner error.

7. ⚠ **C7** (Page 5 (Figure 1 caption) vs Page 4 (§3.1): timing claim)

- **Claim:** Information Horizon consistently occurs early: Figure 1 caption says $t < 1\,\mathrm{s}$; Results give range 0.60 s to 0.92 s.
- **Checks:** inequality_consistency
- **Verdict:** UNCERTAIN
- **Notes:** Planned check: verify $0.92 < 1.0$. Not executed due to runner error.

8. ⚠ **C8** (Page 4 (§3.2) vs Page 6 (Figure 2 caption): transient-phase timing)

    - **Claim:** Sensitivity is highest during the initial transient phase, described as typically within the first two seconds / ($t < 2\,\mathrm{s}$).
    - **Checks:** threshold_phrase_consistency
    - **Verdict:** UNCERTAIN
    - **Notes:** Planned check: confirm both thresholds are 2 s. Not executed due to runner error.

Limitations

- Only parsed text was available; no access to underlying numeric datasets for the 20 oscillators, so regressions/correlations and distributional claims cannot be recomputed.
- Figure-based numeric verification is excluded (no plot-pixel extraction), so time-series claims shown only in figures cannot be quantitatively checked.
- The dynamical equations of motion and parameter distributions for the simulated oscillators are not provided, limiting verification of sensitivity derivatives and any physics-based recomputation beyond algebraic identity checks.
- Execution error prevented running the planned checks: "Sandbox policy violation: from-import of 'typing' is not allowed".