Skepthical review: *Dynamical Stability and Information-Theoretic Constraints of the Graviton Condensate Inflationary Phase*

Summary

The paper proposes an alternative to standard inflaton-driven inflation in which the quasi–de Sitter phase is realized as a Bose–Einstein condensate of soft gravitons with occupation number $N \simeq M_{\mathrm{Pl}}^2/H^2$ (Sec. 2.1). The condensate is modeled by an effective evolution equation including a depletion term and a backreaction term sourced by an information “memory burden” Q_{mem} stored in Bogoliubov modes, with memory accumulation $\dot{Q}_{\mathrm{mem}} \approx N_s H$ for N_s particle species (Secs. 2.1–2.3). Inflation ends via “quantum breaking” when $Q_{\mathrm{mem}} \simeq N$ (Secs. 3.1, 3.4). A phase-space/nullcline picture and numerical integrations (Figs. 1–3) are used to argue for an attractor supporting a quasi–de Sitter stage, and the authors derive an information-theoretic bound $N_e N_s \lesssim M_{\mathrm{Pl}}^2/H^2$ (Sec. 3.3) plus a proposed “dynamical selection” relation for $H(N_s)$ based on an ansatz $Q_{\mathrm{mem}}(H) \sim N_s F(M_{\mathrm{Pl}}/H)$ (Secs. 2.4, 3.2). The conceptual direction—connecting de Sitter entropy/species bounds, quantum breaking, and inflationary phenomenology—is interesting and potentially impactful. However, several central ingredients are currently introduced as phenomenological ansätze without sufficient microscopic justification, the dynamical system appears not fully specified/closed (especially regarding whether and how H evolves with N), the stability analysis is presented in a way that seems mathematically inconsistent with the stated ODEs, and the mapping from condensate fluctuations to curvature perturbations (including amplitude/tilt/tensors/non-Gaussianity) is not derived with enough rigor to assess observational viability. Significant clarification, dimensional/consistency fixes, and reproducibility details are needed for the main claims to be reliably evaluated.

Strengths

- Conceptually original framework replacing an inflaton potential with information/entropy-driven dynamics of a graviton condensate, with clear motivation tied to de Sitter entropy, quantum breaking, and species bounds (Sec. 1; Secs. 2.1, 3.3).
- Compact phenomenological dynamical setup for $N(t)$ and $Q_{\mathrm{mem}}(t)$ with an intuitive physical narrative (depletion vs. memory backreaction) and a clear exit criterion $Q_{\mathrm{mem}} \simeq N$ (Sec. 2.1; Secs. 3.1, 3.4).
- The scaling logic behind the information/species bounds is internally consistent at the order-of-magnitude level (e.g., Eq. (5) and Eq. (6) as scaling relations) and is supported qualitatively by the parameter-space “corridor” plots (Sec. 3.3; Figs. 2–3).
- Systematic exploration of the proposed selection relation $Q_{\mathrm{mem}}(H) \sim N_s F(M_{\mathrm{Pl}}/H)$, including power-law cases $F(x) = x^{\delta}$ and discussion of sub-holographic vs. holographic scaling (Secs. 2.4, 3.2).

- An effort to connect the scenario to cosmological observables (Δ^2_ζ, n_s, r, non-Gaussianity) and to state falsifiable claims rather than purely qualitative speculation (Sec. 2.2; Secs. 3.1, 3.4; Sec. 4).

Major issues

1. **The central dynamical equation for the condensate (Eq. (1) and/or Eq. (3): $\dot{N} = -H + \gamma Q_{\text{mem}}/N^2$) is not dimensionally consistent as written if N and Q_{mem} are dimensionless occupation numbers (Sec. 2.1; Sec. 3.1).** In that case $\dot{N}$ and H have units of time^{-1}, but $\gamma Q_{\text{mem}}/N^2$ is dimensionless unless γ carries units, contradicting the text's description of γ as "order-one constant". This ambiguity propagates into stability, timescales, and numerical integration.

   Recommendation: Make the time variable and normalization explicit (e.g., specify whether derivatives are with respect to cosmic time t, conformal time, or e-fold time $N_e \equiv \ln a$). If t is cosmic time, modify the backreaction term to include an explicit rate scale (typically H or another microscopic rate), or state that γ has dimensions and define it. Then re-check all downstream relations that depend on Eq. (1)/(3) (nullcline condition, eigenvalues in Sec. 3.1, and numerical units in Figs. 1–3).

2. **The dynamical system is not fully specified/closed: the paper asserts $N = M^2_{\text{Pl}}/H^2$ (Sec. 2.1) while also treating N as a dynamical variable evolved by Eq. (1)/(3).** It is unclear whether H is (i) held fixed, (ii) updated algebraically at each time step via $H(N) = M_{\text{Pl}}/\sqrt{N}$, or (iii) evolved by an additional equation. Several claims require consistent $H(t)$ evolution (e.g., "slow drift causes gradual decrease in H" in Sec. 3.1; and $N_e = \int H\,dt$ in Sec. 3.3).

   Recommendation: State explicitly (in Sec. 2.1 and in the numerical-method description) which variables are independent and what equations are solved. If H is derived from N, rewrite the ODE(s) in a manifestly closed form (e.g., $\dot{N} = f(N, Q)$, $\dot{Q} = g(N, Q)$ with $H(N)$ substituted). If H is treated as approximately constant, state the approximation regime and quantify the allowed drift. Ensure the definitions of N_e, tilt, and quantum-breaking time use the same choice.

3. **The stability/attractor analysis in Sec. 3.1 appears mathematically inconsistent with the stated evolution laws and with the terminology used.** With $\dot{Q}_{\text{mem}} \approx N_s H$ (Sec. 2.1), the full 2D system (N, Q_{mem}) generically has no fixed point because $\dot{Q}_{\text{mem}} \neq 0$. What can exist is an attracting nullcline/slow manifold (e.g., $\dot{N} \simeq 0$) with drift along it as Q_{mem} grows. Relatedly, the reported eigenvalue $\lambda = -2H$ (Sec. 3.1, p.5) does not follow from the written equations under standard interpretations (even before accounting for $H(N)$).

   Recommendation: Revise Sec. 3.1 to analyze the correct object: attraction transverse to the $\dot{N} = 0$ nullcline (or a slow manifold if a timescale hierarchy is assumed), rather than a fixed-point Jacobian unless an actual fixed point is introduced. Write the explicit Jacobian of the system you actually integrate, specify whether derivatives are taken at constant H or with $H(N)$, and show the derivation of the transverse eigenvalue(s). If $\lambda = -2H$ is obtained only after nondimensionalization (e.g., using e-fold time), state that clearly and adjust notation accordingly.

4. **Ambiguity between the quasi-static/nullcline condition and the quantum-breaking condition: the paper defines quasi-static equilibrium/attractor behavior via $\dot{N} \approx 0$ (Sec. 2.1), but later uses "equilibrium condition $Q_{\text{mem}} \sim N$" to derive the dynamical selection equation (Sec. 2.4; Sec. 3.2).** Earlier, $Q_{\text{mem}} = N$ is the termination (quantum breaking) criterion (Secs. 2.1–2.3, 3.4), not the condition defining the inflationary attractor.

   Recommendation: Disambiguate and consistently name: (i) the attractor/nullcline condition from $\dot{N} = 0$ (which gives a relation between Q_{mem} and N given H), versus (ii) the breaking/saturation criterion $Q_{\text{mem}} = N$. If the selection mechanism relies on the breaking condition rather than the inflationary attractor, explain why an end-of-phase condition determines the inflationary scale during the phase, and under what dynamical assumptions this is valid.

5. **The derivation of cosmological perturbations is currently too heuristic to assess observational viability.** The key asserted chain $\zeta \sim \delta N/N \Rightarrow \Delta_\zeta^2 \sim 1/N \sim H^2/M_{\text{Pl}}^2$ (Sec. 2.2; Sec. 3.1) is not derived in a gauge-invariant perturbation framework, and it is unclear how graviton-occupation fluctuations map to curvature perturbations (in standard inflation, ζ is related to fluctuations in the local expansion history δN_{efolds}, not directly to a graviton number). Additionally, the numerical statement that $A_s \simeq 2.1 \times 10^{-9}$ is reproduced for $H/M_{\text{Pl}} \sim 10^{-5}$ is inconsistent with $\Delta_\zeta^2 \sim (H/M_{\text{Pl}})^2$ without an extra $\mathcal{O}(10)$ prefactor (Sec. 3.1). Claims about tilt, tensors r, and non-Gaussianity are also not backed by explicit formulas (Secs. 3.1, 3.4; Sec. 4).

   Recommendation: Provide a concrete perturbation derivation: specify the "clock"/single-degree-of-freedom controlling adiabatic perturbations; relate δN (occupation) to $\delta H/H$, $\delta\rho$, or δN_{efolds} in a separate-universe/δN-formalism sense; and show how the two-point function gives Δ_ζ^2. State the statistical assumption for δN (Poissonian? condensate correlator?) and the evaluation/freeze-out condition (is it still $k = aH$? what is c_s and how is it computed?). Give explicit expressions (even parametric) for A_s, $n_s - 1$, r, and a leading estimate of f_{NL}, and reconcile the stated H value with A_s by including the missing prefactors or a model-specific suppression/enhancement mechanism.

6. **The information-theoretic bound $N_e N_s \lesssim M_{\text{Pl}}^2/H^2$ (Sec. 3.3; Eq. (6)) is plausible as a scaling estimate but its derivation assumes approximations that conflict with other statements in the paper.** The step $N_e = Ht_{\text{qb}}$ with $t_{\text{qb}} \sim N/(N_s H)$ assumes H and N are roughly constant while Q_{mem} grows, whereas else-

where the model relies on a slow drift in H along the attractor to generate a red tilt (Sec. 3.1). The relation to the species bound Eq. (5) is also asserted rather than clearly separated (are these independent constraints or two faces of the same cutoff logic?).

Recommendation: Either (i) justify the constant-H/constant-N approximation quantitatively (e.g., show drift is small over the quantum-breaking time in the viable corridor), or (ii) re-derive the bound using $N_e = \int H(t)dt$ with the explicit $H(N(t))$ relation and the actual $\dot{N}, \dot{Q}$ equations. Clarify which assumptions are required for Eq. (5) (species cutoff) versus Eq. (6) (memory/quantum-breaking) and whether one implies the other in your setup.

7. **The "dynamical selection" of H from N_s depends critically on an ad hoc ansatz $Q_{\text{mem}}(H) \sim N_s F(M_{\text{Pl}}/H)$ and on using $Q_{\text{mem}} \sim N$ to infer Eq. (4) (Secs. 2.4, 3.2).** While algebraically consistent (e.g., power-law $F(x) = x^{\delta}$), the model does not provide a microscopic argument for why Q_{mem} should scale as a chosen function of the horizon size, or why the linear case $\delta = 1$ is preferred. The observation that holographic scaling $\delta = 2$ forces $N_s \sim 1$ further underscores that predictions are dominated by the saturation criterion and the choice of F, not derived dynamics.

   Recommendation: Add a dedicated justification (or at least a controlled model) for $Q_{\text{mem}}(H)$: what degrees of freedom are being counted, what sets their density of states, and why does their cumulative "memory" scale as $F(M_{\text{Pl}}/H)$? If a first-principles derivation is not available, demonstrate robustness: show which qualitative outcomes (viable N_e, predicted H, corridor existence) persist across broad families of F and which are artifacts of specific δ. Clearly separate what is a postulate vs. a derived consequence (suggested in Sec. 2.4 and Sec. 3.2).

8. **Conceptual consistency and EFT/energy-budget questions are underdeveloped.** The "memory backreaction pressure" is introduced without an effective stress-energy description or an explicit modification of Einstein's equations, making it unclear how energy conservation and backreaction are implemented consistently (Secs. 2.1–2.2; Sec. 3.4). The reheating claim—efficient energy transfer to N_s species at quantum breaking—is asserted without a mechanism or parametric estimate of transfer/thermalization and homogeneity.

   Recommendation: Clarify the regime of validity (large N, weak coupling, adiabaticity) and provide an effective description of how the memory term sources background evolution (e.g., an effective ρ and p contribution, or a coarse-grained equation consistent with Bianchi identities). For reheating, provide at least parametric estimates of the energy available at breaking, the rate into species, and whether it yields a thermal radiation bath; state any conditions under which reheating is prompt/slow and how this impacts N_e and observables.

9. **Numerical results and the "stability corridor" in (N_s, H) space (Secs. 2.3, 3.1–3.3; Figs. 1–3) are not reproducible from the description.** Key missing items include: the exact ODEs integrated (and whether H is fixed or updated from N), initial conditions for $N(t_0)$ as well as $Q_{\mathrm{mem}}(t_0)$, the definition of e-folds used in the code, solver choice/tolerances, stopping criteria for quantum breaking, and how a scan over "input H" is performed if H is determined by N.

   Recommendation: Add a concise "Numerical methods" subsection (or an appendix) listing the full dynamical system, variable definitions, parameter values (including γ), initial conditions, solver (e.g., RK45/LSODA), tolerances, time/e-fold integration range, event detection for $Q_{\mathrm{mem}} = N$, and how N_e is computed. Ensure the figure captions contain enough detail to reproduce each panel. Consider providing code or pseudocode.

Minor issues

1. Figures 1–3 are difficult to read at typical print scale due to small fonts, dense trajectories (especially Fig. 1), insufficiently distinct color/linestyle choices, and incomplete labeling. Specific issues noted: Fig. 1 lacks explicit axis units/normalizations and clear annotation of initial conditions/parameters per panel; Fig. 2 does not specify the normalization choices for $F(x)$ and does not clearly highlight the $\delta = 2$ case; Fig. 3 lacks a clearly visible colorbar and does not clearly distinguish "allowed" versus "dynamically selected" regions, with missing definitions of functions used in the heatmap (Secs. 3.1–3.3).

   Recommendation: Increase font sizes; reduce trajectory density or show representative trajectories; use colorblind-safe palettes plus redundant line styles/markers; add explicit axis labels with units/normalizations and subpanel letters; annotate initial conditions and key parameter values in captions; add a colorbar and define all plotted quantities directly in figure captions; overlay reference lines for observational targets (e.g., A_s, $N_e \approx 60$) and theoretical bounds (species bound, Eq. (6)).

2. Perturbation-sector quantities are referenced but not defined: e.g., the statement $c_s^2 = \partial p_{\mathrm{mem}} / \partial \rho_{\mathrm{cond}}$ appears without definitions of p_{mem} or ρ_{cond} (Sec. 3.4; Sec. 4).

   Recommendation: Define ρ_{cond}, p_{mem}, and any effective-fluid quantities used, and explain how they are computed from the condensate/memory degrees of freedom. If these are purely parametric placeholders, state that and indicate what microphysics would determine them.

3. The paper would benefit from clearer positioning relative to prior literature on corpuscular gravity/graviton condensates, quantum breaking, de Sitter entropy bounds, and species bounds. The term "memory burden" is evocative but currently under-cited/under-contextualized (Sec. 1; Secs. 2.1, 3.3).

Recommendation: Add citations and a short related-work discussion clarifying which ingredients are adopted from earlier work and which are new here, and define "memory burden" in relation to established information-theoretic or statistical-mechanical constructs (entropy, entanglement, coarse-grained occupation, etc.).

4. Equation repetition and cross-referencing: Eq. (1) and Eq. (3) (and Eq. (2) and Eq. (4)) appear repeated without always clarifying they are restated for convenience, which can confuse references (p.3 vs p.5; p.4 vs p.6).

   Recommendation: Either avoid duplication and reference the earlier equation numbers, or explicitly note when an equation is being restated identically and why (e.g., "reproduced here for convenience").

5. Overuse of $\sim$, $\approx$, and proportionalities without a consistent distinction between (i) definitions/model postulates, (ii) scaling relations, and (iii) late-time/attractor approximations (Secs. 2.1–2.4; Secs. 3.1–3.3).

   Recommendation: Add a short notation/assumption paragraph that labels which relations are exact postulates, which are approximations, and what is meant by "order-one" uncertainties. Where possible, carry explicit $\mathcal{O}(1)$ coefficients or denote them consistently.

Very minor issues

1. Notation and formatting inconsistencies occur across the manuscript (e.g., N_s vs N_s vs N_s rendered as "Ns", $M_{\rm Pl}$ vs M_{Pl}, inconsistent italics/upright symbols), and axes do not always state whether scaling is logarithmic (figures/captions).

   Recommendation: Standardize notation globally; define whether $M_{\rm Pl}$ is reduced Planck mass; standardize tick/scientific notation; explicitly state log/linear axis scalings in captions.

2. Some terminology is introduced without definition in-figure or in caption (e.g., "stability corridor"), and some legends/annotations are too small to be standalone.

   Recommendation: Define key terms either in the main text near first use or directly in figure captions; ensure all legends are readable and self-contained.

3. The author affiliation line ("Anthropic, Gemini & OpenAI servers") is nonstandard for most scientific venues and may be inappropriate depending on submission policies (front matter).

   Recommendation: Replace with standard institutional affiliation(s) or follow the target journal's policy for AI/tool acknowledgments (typically in an acknowledgments or author-contribution statement rather than as an affiliation).

Key statements and references

- • **The consistency of the semiclassical description in the presence of many particle species requires the Hubble scale during inflation to lie below the species-lowered gravitational cutoff, $H \leq M_{\rm Pl}/\sqrt{N_s}$, which implies the species-entropy bound $N_s \leq M_{\rm Pl}^2/H^2 = S_{\rm dS}$, where $S_{\rm dS}$ is the Bekenstein–Hawking entropy of the de Sitter patch.**
- *Reference(s):* $S_{\rm dS}$
- • **The observed amplitude of scalar curvature perturbations, $A_s \approx 2.1 \times 10^{-9}$, is reproduced in this graviton-condensate framework by the relation $\Delta_\zeta^2 \sim 1/N = H^2/M_{\rm Pl}^2$, which fixes the inflationary scale to $H/M_{\rm Pl} \sim 10^{-5}$.**
- *Reference(s):* $A_s \approx 2.1 \times 10^{-9}$
- • **For a linear memory-load scaling $F(x) = x$ (i.e., $\delta = 1$) in the selection equation $N_s F(M_{\rm Pl}/H) \sim M_{\rm Pl}^2/H^2$, the model predicts the simple relation $N_s \sim M_{\rm Pl}/H$, so that the observationally favored inflationary scale $H/M_{\rm Pl} \sim 10^{-5}$ corresponds to a particle content $N_s \sim 10^5$, a value independently motivated by Grand Unified Theories (GUTs).**
- *Reference(s):* Grand

Unified Theories (GUTs)

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: substantial

The paper's central claims rely on a coupled dynamical system for graviton occupation number N and a memory burden $Q_{\rm mem}$, a holographic relation $N = M_{\rm Pl}^2/H^2$, and several derived constraints/selection relations (notably $N_e \cdot N_s \leq M_{\rm Pl}^2/H^2$ and $N_s F(M_{\rm Pl}/H) \sim M_{\rm Pl}^2/H^2$). Several algebraic consequences of these relations are consistent; however, the core evolution equation for N is dimensionally inconsistent as written, and the reported linear stability eigenvalue is not derivable from the provided equations without additional (unstated) assumptions or rescalings. There is also a conceptual/definitional inconsistency in calling $Q_{\rm mem} \sim N$ an "equilibrium condition" despite earlier defining it as the quantum-breaking threshold.

Checked items

1. ✔ **Holographic occupation relation** (Sec. 2.1, p.3 (text: $N = M_{\rm Pl}^2/H^2$))
    - **Claim:** The graviton number satisfies $N = M_{\rm Pl}^2/H^2$.
    - **Checks:** dimensional/units, symbol consistency
    - **Verdict:** PASS; confidence: high; impact: moderate

   - **Assumptions/inputs:** M_{Pl} is the reduced Planck mass, H has units of $1/\mathbf{time}$, N is a dimensionless occupation number
   - **Notes:** Dimensionally consistent in natural units (M_{Pl} and H both inverse length/time), yielding dimensionless N. Used consistently later to rewrite $1/N = H^2/M_{\mathrm{Pl}}^2$.

2. ✖ **Condensate evolution equation units** (Eq. (1), Sec. 2.1, p.3 (also Eq. (3), Sec. 3.1, p.5))
   - **Claim:** $dN/dt = -H + \gamma Q_{\mathrm{mem}}/N^2$, with γ order-one.
   - **Checks:** dimensional/units
   - **Verdict:** FAIL; confidence: high; impact: critical
   - **Assumptions/inputs:** N, Q_{mem} are occupation numbers (dimensionless), t is physical time, H has units $1/\mathbf{time}$, γ is dimensionless per 'order-one constant' language
   - **Notes:** LHS has units $1/\mathbf{time}$. The $-H$ term has units $1/\mathbf{time}$. The backreaction term $\gamma Q_{\mathrm{mem}}/N^2$ is dimensionless under the paper's descriptions unless γ has hidden units of $1/\mathbf{time}$ or time is rescaled. This undermines subsequent stability and timescale statements derived from this ODE.

3. ✔ **Memory accumulation rate units** (Sec. 2.1, p.3 (text: $dQ_{\mathrm{mem}}/dt \approx N_s H$))
   - **Claim:** Memory load grows at rate $dQ_{\mathrm{mem}}/dt \approx N_s H$.
   - **Checks:** dimensional/units
   - **Verdict:** PASS; confidence: high; impact: moderate
   - **Assumptions/inputs:** Q_{mem} dimensionless, N_s dimensionless species count, H has units $1/\mathbf{time}$
   - **Notes:** RHS has units $1/\mathbf{time}$ as required.

4. ⚠ **Nullcline condition from $dN/dt \approx 0$** (Sec. 2.1, p.3 and Sec. 3.1, p.5 (nullcline defined by $dN/dt = 0$ using Eq. (1)/(3)))
   - **Claim:** Near-equilibrium inflation corresponds to $dN/dt \approx 0$ (a nullcline/attractor manifold).
   - **Checks:** algebra, definition consistency
   - **Verdict:** UNCERTAIN; confidence: low; impact: moderate
   - **Assumptions/inputs:** Eq. (1)/(3) is the governing equation for N, H and γ treated as given on the timescale of relaxation
   - **Notes:** While defining a nullcline by $dN/dt = 0$ is standard, the explicit nullcline relation $Q_{\mathrm{mem}} = HN^2/\gamma$ depends on Eq. (1)/(3), which is dimensionally inconsistent as written. Therefore downstream uses of the nullcline are not verifiable without a corrected ODE.

5. ✔ **Quantum breaking threshold definition** (Sec. 2.1, p.3 (text: condition $Q_{\mathrm{mem}} = N$))

  - **Claim:** Inflation ends when Q_{mem} reaches capacity: $Q_{\text{mem}} = N$.
  - **Checks:** definition consistency
  - **Verdict:** PASS; confidence: medium; impact: moderate
  - **Assumptions/inputs:** Q_{mem} and N are comparable measures of capacity/occupation
  - **Notes:** As a model postulate/termination criterion, this is internally consistent with later use in deriving t_{qb} and Eq. (4)/(6). The paper later calls this an "equilibrium condition," which is a terminology inconsistency handled separately.

6. ✔ **Selection equation from saturation + scaling ansatz** (Eq. (2), Sec. 2.4, p.4 (also Eq. (4), Sec. 3.2, p.6))

   - **Claim:** From $Q_{\text{mem}}(H) \sim N_s F(M_{\text{Pl}}/H)$ and $Q_{\text{mem}} \sim N$ with $N = M_{\text{Pl}}^2/H^2$, one gets $N_s F(M_{\text{Pl}}/H) \sim M_{\text{Pl}}^2/H^2$.
   - **Checks:** algebra, symbol consistency
   - **Verdict:** PASS; confidence: high; impact: moderate
   - **Assumptions/inputs:** Saturation/threshold condition $Q_{\text{mem}} \sim N$, Scaling ansatz $Q_{\text{mem}}(H) \sim N_s F(M_{\text{Pl}}/H)$, Holographic relation $N = M_{\text{Pl}}^2/H^2$
   - **Notes:** Direct substitution yields the stated relation.

7. ✔ **Power-law scaling consequences for $\delta = 2$ and $\delta = 1$** (Sec. 3.2, p.6 (discussion under Eq. (4)))

   - **Claim:** If $F(x) = x^\delta$: $\delta = 2 \Rightarrow N_s \sim 1$ (degenerate), $\delta = 1 \Rightarrow N_s \sim M_{\text{Pl}}/H$.
   - **Checks:** algebra, limiting/sanity case
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** Eq. (4): $N_s(M_{\text{Pl}}/H)^\delta \sim M_{\text{Pl}}^2/H^2$
   - **Notes:** Rearranging gives $N_s M_{\text{Pl}}^{\delta-2} = H^{\delta-2}$; for $\delta = 2$, $N_s \sim 1$; for $\delta = 1$, $N_s \sim M_{\text{Pl}}/H$.

8. ✔ **Species cutoff to species-entropy bound** (Eq. (5), Sec. 3.3, p.7)

   - **Claim:** From $H \leq M_{\text{Pl}}/\sqrt{N_s}$ one gets $N_s \leq M_{\text{Pl}}^2/H^2$.
   - **Checks:** algebra, dimensional/units
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** Cutoff inequality $H \leq M_{\text{Pl}}/\sqrt{N_s}$
   - **Notes:** Squaring and rearranging yields $N_s \leq M_{\text{Pl}}^2/H^2$. Identification with S_{dS} is a naming/interpretation step not checked beyond symbol consistency.

9. ✔ **Quantum breaking time from memory growth** (Sec. 3.3, p.7 (text: $t_{\text{qb}} \sim N/(N_s H)$))

   - **Claim:** Given $dQ_{\text{mem}}/dt \sim N_s H$ and breaking at $Q_{\text{mem}} \sim N$, the lifetime is $t_{\text{qb}} \sim N/(N_s H)$.

  - **Checks:** algebra, assumption sufficiency
  - **Verdict:** PASS; confidence: medium; impact: moderate
  - **Assumptions/inputs:** H approximately constant over the accumulation period (or interpreted as an order-of-magnitude rate), N is treated as the relevant capacity scale at breaking
  - **Notes:** Integrating $dQ/dt \approx N_s H$ gives $Q \approx N_s Ht$; setting $Q \approx N$ yields $t \approx N/(N_s H)$. If $H(t)$ varies appreciably, a time integral is needed; the paper does not show that step.

10. ✔ **Information-theoretic e-fold bound** (Eq. (6), Sec. 3.3, p.7)
    - **Claim:** $N_e \cdot N_s \leq M_{\rm Pl}^2/H^2$.
    - **Checks:** algebra, assumption sufficiency
    - **Verdict:** PASS; confidence: medium; impact: critical
    - **Assumptions/inputs:** $N_e = Ht_{\rm qb}$ (or $N_e = \int H dt$ approximated by $Ht_{\rm qb}$), $t_{\rm qb} \sim N/(N_s H)$, $N = M_{\rm Pl}^2/H^2$ at the relevant epoch
    - **Notes:** Combining gives $N_e \approx H \cdot N/(N_s H) = N/N_s$, hence $N_e N_s \approx N = M_{\rm Pl}^2/H^2$. The paper writes $\leq$, consistent with order-of-magnitude/upper-bound intent. If H varies, the bound needs a derivation using $N_e = \int H dt$.

11. ✖ **Stability eigenvalue $\lambda = -2H$ claim** (Sec. 3.1, p.5 (text: Jacobian yields $\lambda = -2H$; $t_{\rm decay} \sim 1/(2H)$))
    - **Claim:** Linearization around equilibrium gives a strongly negative eigenvalue $-2H$ in the N-direction.
    - **Checks:** algebra, definition consistency
    - **Verdict:** FAIL; confidence: medium; impact: critical
    - **Assumptions/inputs:** State vector includes at least N and $Q_{\rm mem}$, Dynamics governed by Eq. (3) and $dQ_{\rm mem}/dt \approx N_s H$, Equilibrium defined by $dN/dt = 0$
    - **Notes:** No Jacobian is shown. Under the simplest interpretation with H treated constant, $\partial(dN/dt)/\partial N|{\rm eq} = -2\gamma Q/N^3 = -2H/N(usingequilibriumrelation\gamma Q/N^2=H), not-2H.IfH dependsonNviaN=M^2/H^2, additionaltermsariseandstilldonotyield-2H$ without further assumptions/rescalings that are not provided.

12. ⚠ **Curvature perturbation mapping and spectrum scaling** (Sec. 2.2, p.3 and Sec. 3.1, p.5 ($\zeta \sim \delta N/N$; $\Delta\zeta^2 \sim 1/N = H^2/M_{\rm Pl}^2$))
    - **Claim:** ζ is proportional to $\delta N/N$ and the power spectrum amplitude scales as $1/N$, hence $\Delta\zeta^2 \sim H^2/M_{\rm Pl}^2$.
    - **Checks:** symbol consistency, missing-steps audit
    - **Verdict:** UNCERTAIN; confidence: low; impact: moderate

    - **Assumptions/inputs:** $\zeta \sim \delta N/N$, $\mathrm{Var}(\delta N)$ scaling is such that $\mathrm{Var}(\delta N)/N^2 \sim 1/N$
    - **Notes:** The identity $1/N = H^2/M_{\mathrm{Pl}}^2$ follows from $N = M_{\mathrm{Pl}}^2/H^2$ and is consistent. However, the crucial step $\Delta\zeta^2 \sim 1/N$ from $\zeta \sim \delta N/N$ requires an explicit assumption about the statistics/magnitude of δN that is not provided.

13. ⚠ **Speed of sound definition and $c_s \approx 1$ claim** (Sec. 2.2, p.3 ($c_s^2 = \partial p_{\mathrm{mem}}/\partial \rho_{\mathrm{cond}}$) and Sec. 3.1, p.5 ($c_s^2 \approx 1$))
    - **Claim:** Memory burden acts as a pressure giving c_s^2 defined by $\partial p_{\mathrm{mem}}/\partial \rho_{\mathrm{cond}}$ and yielding $c_s^2 \approx 1$.
    - **Checks:** missing-steps audit, definition sufficiency
    - **Verdict:** UNCERTAIN; confidence: low; impact: minor
    - **Assumptions/inputs:** Well-defined $p_{\mathrm{mem}}(\cdot)$ and $\rho_{\mathrm{cond}}(\cdot)$ exist in the model
    - **Notes:** No analytic expressions for p_{mem} or ρ_{cond} are given, so the derivative and the conclusion $c_s^2 \approx 1$ cannot be checked symbolically.

Limitations

- Audit is based only on the provided 10 pages of extracted PDF text/images; no appendices or intermediate derivations were available.
- No numerical/simulation outputs were checked (by request); only symbolic consistency of stated formulas and their immediate algebraic consequences was assessed.
- Several key claims (Jacobian/eigenvalues, fluctuation amplitude, speed of sound) are asserted without shown derivations; these were marked UNCERTAIN/FAIL depending on whether they contradict the stated equations.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

10 candidate numerical/algebraic checks were executed: 4 PASS and 6 FAIL under the provided tolerances. The main substantive numeric tension is the scalar amplitude scaling example (ratio 21 between stated A_s and $(H/M_{\mathrm{Pl}})^2$). Several other FAIL results appear to stem from enforcing exact zero tolerance in floating-point identity evaluations.

Checked items

1. ✖ **C1** (Page 3, Sec. 2.1)
    - **Claim:** Holographic relation: $N = M_{\mathrm{Pl}}^2/H^2$.
    - **Checks:** algebraic_rearrangement_consistency
    - **Verdict:** FAIL
    - **Notes:** Identity check failed under numeric substitution (unexpected).

2. ✖ **C2** (Page 5, Sec. 3.1)
    - **Claim:** Power spectrum amplitude relation: $\Delta_\zeta^2 \sim 1/N = H^2/M_{\rm Pl}^2$.
    - **Checks:** derived_identity_from_defined_relation
    - **Verdict:** FAIL
    - **Notes:** Identity check failed under numeric substitution (unexpected).
3. ✔ **C3** (Page 5, Sec. 3.1)
    - **Claim:** Claim: $A_s \approx 2.1 \times 10^{-9}$ is reproduced for $H/M_{\rm Pl} \sim 10^{-5}$ using $\Delta_\zeta^2 \sim H^2/M_{\rm Pl}^2$.
    - **Checks:** order_of_magnitude_numeric_check
    - **Verdict:** PASS
    - **Notes:** Computed predicted_As=$(H/M_{\rm Pl})^2 = 1.0000000000000002 \times 10^{-10}$ and ratio A_s/pred=21.0.
4. ✔ **C4** (Page 5, Sec. 3.1)
    - **Claim:** Eigenvalue $\lambda = -2H$ implies decay timescale $t_{\rm decay} \sim 1/(2H)$.
    - **Checks:** timescale_from_eigenvalue
    - **Verdict:** PASS
    - **Notes:** Checked definition $t_{\rm decay} = 1/|\lambda|$ with $\lambda = -2H$ for $H > 0$.
5. ✔ **C5** (Page 7, Eq. (5) and surrounding text)
    - **Claim:** From cutoff $H \leq M_{\rm Pl}/\sqrt{N_s}$, the species bound $N_s \leq M_{\rm Pl}^2/H^2$.
    - **Checks:** inequality_rearrangement
    - **Verdict:** PASS
    - **Notes:** Tested equivalence numerically on random positive samples (inequalities can be safely squared under positivity).
6. ✖ **C6** (Page 7, Sec. 3.3)
    - **Claim:** Given $dQ_{\rm mem}/dt \sim N_s H$ and threshold $Q_{\rm mem}(t_{\rm qb}) \sim N$, maximum lifetime $t_{\rm qb} \sim N/(N_s H)$.
    - **Checks:** integration_of_constant_rate
    - **Verdict:** FAIL
    - **Notes:** Verified constant-rate integration with $Q(0) = 0$.
7. ✖ **C7** (Page 7, Eq. (6) derivation)
    - **Claim:** With $N_e = H t_{\rm qb}$ and $t_{\rm qb} \sim N/(N_s H)$, get $N_e \cdot N_s \leq M_{\rm Pl}^2/H^2$.
    - **Checks:** chain_derivation_consistency
    - **Verdict:** FAIL
    - **Notes:** Identity check failed under numeric substitution (unexpected).
8. ✖ **C8** (Page 6-7, Sec. 3.2 (linear scaling $\delta = 1$))

  - **Claim:** For linear scaling ($\delta = 1$), selection equation yields $N_s \sim M_{\mathrm{Pl}}/H$; thus $H/M_{\mathrm{Pl}} \sim 10^{-5}$ implies $N_s \sim 10^5$.
  - **Checks:** plug_in_numeric_consistency
  - **Verdict:** FAIL
  - **Notes:** Computed $N_{s,\mathrm{pred}} = 99999.99999999999$ vs claimed 100000.0; mismatch is at floating-point roundoff level.

9. ✔ **C9** (Page 8, Sec. 3.3)

   - **Claim:** Claim: For $H/M_{\mathrm{Pl}} \sim 10^{-5}$, model can accommodate up to $N_s \sim 10^8$ while still achieving over 60 e-folds.
   - **Checks:** constraint_feasibility_check
   - **Verdict:** PASS
   - **Notes:** Computed $N_{e,\mathrm{max}} = 99.99999999999999$, which exceeds 60.

10. ✔ **C10** (Page 4, Sec. 2.3)

    - **Claim:** Simulation grid: $N_s \in [1, 10^8]$ and $H/M_{\mathrm{Pl}} \in [10^{-5}, 10^{-2}]$.
    - **Checks:** range_endpoint_sanity_and_ordering
    - **Verdict:** PASS
    - **Notes:** Ordering holds; spans 8 decades in N_s and 3 decades in H/M_{Pl}.

Limitations

- Checks are based only on the provided parsed text; no additional equations/appendices/tables beyond these pages were available.
- No validation of numerical simulations, phase portraits, or heatmaps is attempted because it would require reproducing integrations or extracting values from figures.
- Where the paper uses proportionalities ($\sim$) and order-one constants (e.g., γ), only algebraic consistency and rough scaling checks are feasible; exact numerical agreement cannot be confirmed without missing prefactors.
- Several checks were evaluated with zero absolute/relative tolerance (abs_tol=0, rel_tol=0) and failed by very small floating-point-level differences in some cases; this limits interpretability of PASS/FAIL for exact identities under numeric substitution.