Skepthical review: *Transient Superdiffusion in Forced Two-Dimensional Turbulence: A Crossover Phenomenon Governed by Restorative Correlations*

Summary

The manuscript studies Lagrangian dispersion of passive tracers in forced 2D incompressible turbulence using DNS, with the goal of disentangling whether reported superdiffusion is (i) a genuine asymptotic anomalous-transport property (e.g., due to inverse-cascade long-range correlations, L\'evy/CTRW mechanisms, or intermittent strain-guided “highways” between vortex trapping events) or (ii) a long finite-time crossover from early-time ballistic motion toward normal diffusion. The authors analyze mean-squared displacement (MSD) and a running Hurst exponent $H(t)$ (Sec. 2.3, Sec. 3.1), condition trajectories on the Okubo--Weiss (OW) strain/vortex partition via the fraction of time spent in strain (Sec. 2.2, Sec. 3.2), and compare to phase-randomized surrogate Lagrangian velocity time series meant to preserve spectral content while destroying phase coherence (Sec. 2.4, Sec. 3.3). They report that (a) $H(t)$ decays from near-ballistic values toward ~ 0.56 at the latest accessible times ($t \lesssim 600$), consistent with a slow crossover; (b) “strain-dominated” and “vortex-dominated” subpopulations (defined by $> 70\%$ vs $< 30\%$ time in strain) exhibit nearly identical MSD/$H(t)$, arguing against a distinct highway population; (c) surrogates disperse much faster (near-ballistic), which is interpreted as evidence for restorative/anti-persistent temporal structure in the true dynamics; and (d) increment-tail and trapping-time statistics do not support L\'evy-flight/CTRW explanations (Sec. 3.4). The questions are timely and the OW-conditioning + surrogate comparison is promising, but several conclusions are currently stronger than what the documented methods/time range can robustly support: essential DNS details for reproducibility/regime identification are missing (Sec. 2.1), OW conditioning appears under-validated given sparse Eulerian sampling (Sec. 2.2), the “fraction-of-time-in-strain” diagnostic may not capture intermittent flight-driven transport (Sec. 3.2), the surrogate construction/interpretation contains conceptual inaccuracies (Sec. 2.4--3.3), and uncertainty/robustness quantification for late-time scaling is insufficient (Sec. 3.1--4).

Strengths

- Addresses a clear and relevant mechanistic question in 2D turbulence transport (inverse-cascade correlations vs strain “highways” vs L\'evy/CTRW), with a coherent narrative from Secs. 1--4.
- Uses a conceptually direct OW-based flow partition (Sec. 2.2) and tracer classification to operationalize and test the “highway” hypothesis (Sec. 3.2).
- Employs a running Hurst-exponent diagnostic (Eq. (3), Sec. 2.3) that makes the ballistic-to-diffusive crossover behavior visible rather than relying on single power-law fits.

- The surrogate-trajectory idea (Sec. 2.4, Sec. 3.3) is potentially very informative for isolating which aspects of temporal structure (beyond the spectrum) influence dispersion.
- Attempts to rule out alternative anomalous-transport explanations (L\'evy/CTRW) using increment and trapping-time statistics (Sec. 2.3, Sec. 3.4), which is valuable if made more quantitative/transparent.

Major issues

1. **DNS configuration and turbulence-regime identification are insufficiently documented (Sec. 2.1), preventing reproducibility and making it hard to assess whether the simulated flow corresponds to a well-developed inverse-cascade regime (and how general the conclusions are).** Key missing items include: domain size/geometry and boundary conditions; numerical method (e.g., pseudo-spectral vs finite difference, de-aliasing); grid resolution; time integrator and time step; forcing definition (stochastic vs deterministic, correlation time, amplitude, injection rate, forcing wavenumber band); and large-scale energy control (e.g., linear friction/hypoviscosity) and whether a condensate forms. Also missing are standard diagnostics confirming the intended regime (energy spectrum slopes, fluxes, stationarity).

   Recommendation: Substantially expand Sec. 2.1 (or add an appendix) to specify: (i) domain, BCs, and nondimensionalization; (ii) discretization, de-aliasing, resolution, and timestepper; (iii) forcing form and statistics including injection rates; (iv) any large-scale friction/hypoviscosity and how condensate/box-scale growth is controlled; and (v) regime diagnostics (time-averaged spectra and energy/enstrophy fluxes) demonstrating the inverse cascade and statistical stationarity. This will also help interpret why the accessible time window ($t \leq 600$) is/was not long relative to large-eddy times.

2. **The central interpretation ("superdiffusion is transient; $H(t)$ is slowly relaxing toward 0.5") is not yet statistically secured given the limited late-time range and lack of uncertainty quantification (Sec. 2.3, Sec. 3.1, Sec. 4).** With $t \leq 600$ and a sliding window ($\Delta t = 100$), the late-time estimate $H \approx 0.56$ could still be compatible with: (i) an eventual plateau at weak superdiffusion ($H > 0.5$), (ii) continued decay to 0.5, or (iii) biases from windowing/finite-length effects and limited independent samples.

   Recommendation: In Sec. 3.1 (and echoed in Sec. 4), add robustness/uncertainty quantification for MSD and $H(t)$: (i) bootstrap or block-bootstrap confidence bands over tracers and over time origins t_0; (ii) sensitivity of $H(t)$ to window width (e.g., $\Delta t = 50/100/200$) and the fitting range in Eq. (3); (iii) compensated plots such as $MSD(\tau)/\tau$ (diffusive compensation) and $MSD(\tau)/\tau^{2H_{\text{fit}}}$ over the final decade to visualize convergence; and (iv) a simple model comparison on the late-time segment (constant plateau vs slowly decaying $H(t)$) with uncertainties. If longer runs or multiple realizations are available, extend the time range or ensemble; otherwise, soften Sec. 4 wording to “consistent with a slow approach toward normal diffusion over accessible times.”

3. **Okubo--Weiss (OW) conditioning may be unreliable as implemented due to sparse Eulerian sampling (15 snapshots) and potentially ambiguous partitioning using only sign(Q) (Sec. 2.2, Sec. 3.2).** With ~ 40 time-unit snapshot spacing (if uniform over 600), linear interpolation of $Q(x,t)$ may miss rapid evolution near hyperbolic regions and vortex boundaries; misclassification typically homogenizes conditional statistics, which could mask true subpopulation differences relevant to “highways.”

   Recommendation: In Sec. 2.2, report snapshot spacing and compare it to relevant time scales (forcing-scale turnover, typical vortex orbital/trapping times, Lagrangian velocity correlation time). Where possible, recompute OW at higher cadence for a subset of the run and quantify classification stability (fraction of particles changing class; changes in residence-time PDFs; changes in conditional MSD/$H(t)$). Also test more robust OW criteria: apply a threshold $|Q| > Q_0$ (e.g., Q_0 based on Q_{rms} or local strain/vorticity scales) to exclude ambiguous $Q \approx 0$ regions, and/or use normalized OW. Document how Q is computed (velocity recovery, differentiation, filtering) and how Q is interpolated to particle positions (Sec. 2.2).

4. **The chosen diagnostic for “highways”---classifying particles by total fraction of time spent in strain ($> 70\%$ vs $< 30\%$)---may not detect intermittent flight-dominated transport (Sec. 2.2, Sec. 3.2).** A particle could spend little time in strain yet acquire most of its net displacement during rare, fast strain-guided segments; conversely, long residence in strain does not guarantee large net displacement if motion frequently turns/cancels.

   Recommendation: Augment Sec. 3.2 with event-/segment-based diagnostics closer to the “highway” mechanism: (i) condition instantaneous or short-lag displacement/velocity statistics on being in strain at the current time ($Q > 0$, or $|Q| > Q_0$) rather than global occupancy; (ii) decompose MSD growth into contributions from strain vs vortex segments (e.g., via conditioned velocity autocorrelation integrals); (iii) define “flights” between trapping events (or via turning-angle/curvature criteria) and report flight-length and flight-duration distributions and their contribution to MSD; and (iv) test robustness to thresholds/quantile splits (not only $70/30$). This can either strengthen the negative result (“no highway contribution”) or reveal a more subtle mechanism missed by occupancy-based grouping.

5. **Phase-randomized surrogate construction and interpretation contain conceptual and methodological ambiguities (Sec. 2.4, Sec. 3.3, Sec. 4).** As written, the surrogate is described as destroying “all temporal ordering and correlations” and preserving the “exact same velocity PDF,” which is generally incorrect for standard phase randomization. Moreover, if phases are randomized independently per

component/particle, cross-component correlations (u--v), rotational structure, and geometric constraints of 2D incompressible dynamics are destroyed; large MSD differences could therefore reflect loss of kinematic constraints rather than (or in addition to) “restorative temporal correlations.” The reported near-ballistic surrogate scaling across the full duration also needs reconciliation with what statistics are actually preserved (PSD implies preservation of the autocorrelation function under stationarity/Wiener--Khinchin).

Recommendation: Rewrite Sec. 2.4 and the related claims in Sec. 3.3--4 to state precisely which statistics are preserved (Fourier amplitudes/PSD; under stationarity this preserves second-order autocorrelation) and which are destroyed (phase coherence/higher-order temporal structure, intermittency, and potentially cross-component/cross-particle structure). Provide implementation details: detrending/windowing, treatment of finite-length periodicity, whether phases are randomized per component and whether u--v cross-spectra are preserved, and how surrogate trajectories are integrated. In Sec. 3.3, plot original vs surrogate Lagrangian velocity autocorrelation $R_v(\tau)$ (and ideally cross-correlation between components) to support the mechanistic interpretation. If the intent is a “memoryless” null, additionally include a time-shuffled surrogate that preserves the one-point velocity PDF but breaks temporal dependence, and compare outcomes. Temper conclusions to attribute differences to specific destroyed structures (e.g., coherent turning/vortex-induced phase relations) rather than “removal of all correlations.”

6. **The evidence used to rule out L\'evy-flight and CTRW mechanisms is currently under-specified and lacks uncertainty estimates (Sec. 2.3, Sec. 3.4).** Hill-estimator tail exponents β and trapping-time analyses depend strongly on threshold/fit-range choices and on dependence in the data; without methodological transparency and confidence intervals, the negative conclusion (“rules out L\'evy/CTRW”) is not fully supported.

   Recommendation: Expand Sec. 2.3 and Sec. 3.4 to document: (i) the precise tail convention (PDF vs CCDF) and moment criteria under that convention; (ii) how the Hill threshold (number of upper order statistics) is chosen (Hill plots, stability ranges), with confidence intervals (bootstrap/block-bootstrap); (iii) sample sizes and dependence handling; and (iv) vortex-core and trapping-event definitions (Q threshold(s), additional $|\omega|$ criteria, connectivity/size filters, and entry/exit detection). Include empirical CCDFs for increments and trapping times with alternative tail-model comparisons (e.g., power law vs stretched exponential) and revise the wording to “no compelling evidence for L\'evy/CTRW in this dataset” unless the strengthened analysis justifies a stronger exclusion.

7. **Key figures supporting the main claims appear incomplete and/or lack essential uncertainty/robustness information (notably Fig. 1 as referenced in Sec. 3.1--3.2).** The caption indicates an MSD and an $H(t)$ panel, but the $H(t)$ panel

is reported as missing/not visible; curves are compared without confidence bands; and surrogate results are shown without variability across surrogate realizations, making the strength of the contrast hard to gauge.

Recommendation: Ensure Fig. 1 includes both panels with clear (a)/(b) labels matching the caption. Add uncertainty bands for MSD and $H(t)$ (bootstrap over tracers/time origins), and show multiple surrogate realizations (median plus 5--95% envelope, or thin-line overlays). On the $H(t)$ panel, mark the late-time averaging window used for quoted values (e.g., $H \approx 0.56$) and report that value with a CI. These changes directly support the manuscript's primary inferences.

Minor issues

1. The MSD definition around Eq. (2) (Sec. 2.3) is incomplete (sentence ends with "where") and does not clearly state averaging over tracers and time origins t_0; this affects reproducibility and finite-length bias assessment.

   Recommendation: Complete the text after Eq. (2) with an explicit definition, e.g., $MSD(\tau) = \langle |x_i(t_0 + \tau) - x_i(t_0)|^2 \rangle_{i,t_0}$, specify how many t_0 are used for each τ, and state any exclusion/windowing near the record ends.

2. The running Hurst-exponent estimation (Eq. (3), Sec. 2.3) is under-described: $\Delta t = 100$ is not justified, and it is unclear whether the window slides linearly or logarithmically and how edge effects are treated.

   Recommendation: In Sec. 2.3, specify windowing (linear vs log), step size between windows, and edge handling; justify Δt relative to correlation times and total duration; and briefly report how the qualitative trends change with alternative Δt.

3. Okubo--Weiss computation details are incomplete (Sec. 2.2): formulas for s^2 and ω^2 are not written explicitly; how velocity/gradients are obtained from vorticity and what filtering is applied are not specified; interpolation of Q to particle locations is unspecified.

   Recommendation: Add explicit formulas for s^2 and ω (2D scalar vorticity), and describe the computation pipeline (streamfunction inversion, differentiation scheme, filtering/smoothing if any, and interpolation order to particle positions).

4. The operational definition of vortex trapping times (Sec. 2.3, Sec. 3.4) is too high level: it is unclear how vortex cores are identified and how entry/exit events are detected, and whether short visits are filtered.

   Recommendation: Provide a concise algorithm: vortex-core identification ($Q < -Q_0$ and/or $|\omega| > \omega_0$, connectivity/minimum area), how particle residence intervals are constructed (contiguous time steps, interpolation handling), and how visits are counted/merged. Report typical sample sizes.

5. Terminology such as “anti-persistent,” “restorative,” and “memoryless random walk” is used informally and inconsistently with the surrogate method (Sec. 2.4, Sec. 3.3, Sec. 4).

   Recommendation: Define these terms quantitatively when first used (e.g., anti-persistence via negative/oscillatory velocity autocorrelation; “restorative” as reducing MSD growth relative to a specified surrogate), and avoid calling the phase-randomized surrogate “memoryless” unless temporal dependence is demonstrably removed.

6. The Introduction (Sec. 1) would benefit from clearer positioning relative to prior DNS/experimental studies and earlier OW-conditioned or CTRW/L\'evy interpretations.

   Recommendation: Expand Sec. 1 with a short, targeted literature context: key studies reporting superdiffusion in 2D turbulence, those advocating highway dynamics, and those proposing L\'evy/CTRW mechanisms; state explicitly what is new in the present OW-history + surrogate approach.

7. Subpopulation sizes for the $> 70\%$ strain, 30--70% intermediate, and $< 30\%$ strain classes are not reported (Sec. 2.2, Sec. 3.2), limiting assessment of statistical power.

   Recommendation: Report counts/fractions for each class and comment on relative uncertainties (ideally reflected in the proposed confidence bands).

8. Notation for lag time is inconsistent (t vs τ in Eq. (2)--(3) and figures/text), and reference-slope labels in the MSD plot can be confusing because the MSD exponent is $\alpha = 2H$.

   Recommendation: Standardize lag notation (use τ everywhere or define $\tau := t$ explicitly) and label reference slopes as $\alpha = 2$ ($H = 1$, ballistic) and $\alpha = 1$ ($H = 0.5$, diffusive), optionally annotating both α and H.

9. The manuscript sometimes overstates “homogeneity throughout the flow” based only on coarse OW partition comparisons (Sec. 3.2--3.3, Sec. 4).

   Recommendation: Qualify the claim to “no strong difference between OW-defined subpopulations,” and, if feasible, add a simple additional conditional check (e.g., conditioning on $|Q|$ magnitude or proximity to intense vortices) to probe finer heterogeneity.

Very minor issues

1. Scattered typographical/formatting problems: incomplete sentence after Eq. (2); HTML escape sequences (e.g., “$> 70\%$”) in captions; inconsistent spacing around inline math; inconsistent quotation style for “highways”; mixed heading syntax (Sec. 1--3).

Recommendation: Proofread and fix LaTeX/rendering issues: complete the Eq. (2) sentence, replace HTML escapes with LaTeX, standardize math spacing and quotation style, and regularize section headings.

2. Figure accessibility/legibility: reliance on red/green and color-only identification; missing/unclear axis units or nondimensionalization statements; small fonts/line weights.

   Recommendation: Use a colorblind-safe palette plus line styles/markers, label units (or state nondimensionalization) on axes/captions, increase font sizes/line thickness, and keep color/linestyle mapping consistent across figures.

3. The velocity autocorrelation $R_v(\tau)$ is referenced (Sec. 3.3 / Fig. 3 caption) without a precise mathematical definition (vector dot-product vs component-wise, normalization, averaging over i and t_0).

   Recommendation: Add a one-line definition of $R_v(\tau)$ with normalization and averaging procedure.

Key statements and references

- • **One leading hypothesis attributes superdiffusion in two-dimensional turbulence to long-range spatiotemporal correlations in the velocity field that arise from the inverse energy cascade, whereas an alternative theory proposes that particles undergo intermittent trapping in vortices interspersed with rapid, ballistic-like flights along strain-dominated 'highways' connecting these structures, so that spatial intermittency rather than spectral properties drives anomalous transport [1–7].**
- *Reference(s):* [1], [2], [3]
- • **L\'evy-flight models of anomalous diffusion in turbulent flows posit that superdiffusion is generated by velocity increments with heavy-tailed, infinite-variance probability distributions, while Continuous Time Random Walk (CTRW) models require a power-law distribution of trapping times with exponent $\mu < 2$; in contrast, the present data show velocity-increment tails with finite-variance exponents $\beta > 3$ for all lags and no power-law tail in vortex-core trapping times, thereby ruling out these mechanisms as explanations for the observed transport [8–15].**
- *Reference(s):* [8], [9], [10]
- • **Previous studies of two-dimensional turbulence have argued that coherent vortices and strain-dominated filaments create spatially heterogeneous transport pathways, with vortices acting as trapping regions and strain filaments serving as preferential channels for long-range, nearly ballistic particle motion, leading to persistent superdiffusive scaling of the mean squared displacement over extended time ranges [3, 5, 9].**

- *Reference(s):* [3], [5], [9]

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: substantial

The paper's analytic core relies on (i) MSD scaling with a Hurst exponent and a running estimate $H(t)$ via a log-derivative, (ii) partitioning using the Okubo--Weiss parameter $Q = s^2 - \omega^2$, and (iii) a surrogate-data argument based on phase randomization in Fourier space. The basic MSD/Hurst algebra and dimensional consistency check out. The main internal-consistency problems are in the surrogate-data claims: phase randomization is asserted to preserve the single-time velocity PDF and destroy all temporal correlations (memoryless null), but preserving the power spectrum analytically implies preserving second-order temporal correlations (autocorrelation), and the marginal PDF is not generally preserved. These issues affect the logical basis for concluding that the original dynamics are strongly restorative solely from surrogate-vs-original MSD comparisons.

Checked items

1. ✔ **Okubo--Weiss definition** (Eq. (1), Sec. 2.2, p.2)
    - **Claim:** Defines Okubo--Weiss parameter as $Q = s^2 - \omega^2$; $Q > 0$ strain-dominated, $Q < 0$ rotation-dominated.
    - **Checks:** dimensional consistency, symbol/definition consistency
    - **Verdict:** PASS; confidence: medium; impact: minor
    - **Assumptions/inputs:** s^2 and ω^2 are computed consistently from the same velocity-gradient field, ω is the (scalar) 2D vorticity; s^2 is a nonnegative scalar strain measure
    - **Notes:** Units match (both terms scale like $1/\mathrm{time}^2$). However, s^2 and ω are not explicitly defined, so normalization/factor conventions cannot be verified from the PDF text alone.

2. ✔ **MSD definition with time origin** (Eq. (2), Sec. 2.3, p.2)
    - **Claim:** $MSD(t) = \langle |x(t_0 + t) - x(t_0)|^2 \rangle$ as an ensemble average over a population.
    - **Checks:** definition consistency, dimensional consistency
    - **Verdict:** PASS; confidence: high; impact: minor
    - **Assumptions/inputs:** Angle brackets denote an average over the chosen tracer population (and possibly over t_0 if intended)
    - **Notes:** Expression is dimensionally correct and a valid MSD definition. The averaging set (over particles only vs particles and time origins) is not explicitly specified.

3. ✔ **MSD scaling with Hurst exponent** (Sec. 1 (intro) p.1; Sec. 2.3 p.3)
    - ○ **Claim:** Superdiffusion defined by $MSD(t) \sim t^{2H}$ with $H > 0.5$; ballistic corresponds to $H = 1$; normal diffusion $H = 0.5$.
    - ○ **Checks:** algebraic sanity check, limiting cases
    - ○ **Verdict:** PASS; confidence: high; impact: minor
    - ○ **Assumptions/inputs:** Scaling is interpreted as a power law over some time range
    - ○ **Notes:** Ballistic scaling $MSD \propto t^2$ corresponds to $H = 1$; diffusive $MSD \propto t$ corresponds to $H = 0.5$. Internally consistent.

4. ✔ **Running Hurst exponent formula** (Eq. (3), Sec. 2.3, p.3)
    - ○ **Claim:** Defines $H(t) = (1/2)\, d \log MSD(t)/d \log t$.
    - ○ **Checks:** algebra between definitions, consistency with scaling law
    - ○ **Verdict:** PASS; confidence: high; impact: moderate
    - ○ **Assumptions/inputs:** $MSD(t) > 0$ over the range of interest, Derivative is interpreted in a local/log-log slope sense
    - ○ **Notes:** If $MSD(t) = Ct^{2H}$, then $d \log MSD/d \log t = 2H$, so Eq. (3) recovers constant H. Correct factor $1/2$.

5. ✖ **Lag-time notation consistency (t vs τ)** (Eq. (2)--(3) Sec. 2.3 p.2--3 vs Sec. 3.1 p.3 and Fig. 1 caption p.4)
    - ○ **Claim:** Uses t in equations but uses lag time τ in results/figures.
    - ○ **Checks:** notation consistency
    - ○ **Verdict:** FAIL; confidence: high; impact: minor
    - ○ **Assumptions/inputs:** t and τ are intended to denote the same lag variable
    - ○ **Notes:** Equations are written in terms of t (and t_0), while results and plots discuss $MSD(\tau)$ and $R_v(\tau)$. This is likely a notational slip but creates ambiguity in interpretation of $H(t)$ vs $H(\tau)$.

6. ✔ **Velocity increment definition** (Sec. 2.3, p.3)
    - ○ **Claim:** Defines $\delta v(\tau) = v(t + \tau) - v(t)$.
    - ○ **Checks:** definition consistency
    - ○ **Verdict:** PASS; confidence: high; impact: minor
    - ○ **Assumptions/inputs:** v is the Lagrangian velocity time series along a trajectory
    - ○ **Notes:** Standard definition; consistent with later discussion of increment PDFs.

7. ⚠ **Phase-randomization procedure description** (Sec. 2.4, p.3)

    - **Claim:** Constructs surrogate by DFT, randomizing phases uniformly on $[0, 2\pi)$, and inverse DFT.
    - **Checks:** procedural mathematical consistency
    - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
    - **Assumptions/inputs:** DFT/inverse DFT are applied consistently to real-valued time series (implying conjugate-symmetry handling, though not stated)
    - **Notes:** The outlined steps are mathematically coherent, but the paper does not specify how real-valuedness is enforced (conjugate symmetry of Fourier coefficients). Without that detail, the exact surrogate construction cannot be fully verified.

8. ✖ **Claim: surrogate preserves exact velocity PDF** (Sec. 2.4, p.3)

    - **Claim:** States: preserving power spectral density implies surrogate has the exact same velocity probability distribution and single-time variance as the original data.
    - **Checks:** analytic implication check, definition/statistics consistency
    - **Verdict:** FAIL; confidence: high; impact: moderate
    - **Assumptions/inputs:** Only phases are randomized; Fourier amplitudes are retained
    - **Notes:** Preserving the power spectral density directly supports preserving second-order structure tied to the spectrum (including variance), but does not analytically guarantee preserving the marginal (single-time) probability distribution. The “consequently” inference is not justified as written.

9. ✖ **Claim: surrogate destroys all temporal correlations / is memoryless** (Sec. 2.4, p.3; Sec. 3.3, p.5)

    - **Claim:** States: surrogate has all temporal ordering and correlations destroyed and serves as a null model for a memoryless random walk with the same velocity statistics.
    - **Checks:** analytic implication check, internal logic consistency
    - **Verdict:** FAIL; confidence: high; impact: critical
    - **Assumptions/inputs:** Surrogate preserves the original power spectrum exactly
    - **Notes:** If the surrogate preserves the power spectrum, then (at minimum) second-order temporal correlation structure implied by that spectrum is preserved; thus the surrogate cannot be described as having all temporal correlations destroyed or as memoryless in the usual analytic sense. This directly affects the stated interpretation of surrogate-vs-original differences as purely due to temporal correlations.

10. ⚠ **Inference: surrogate comparison proves restorative (anti-persistent) correlations** (Sec. 3.3, p.5)
    - **Claim:** Concludes that suppressed MSD relative to the surrogate provides direct evidence of strongly restorative (anti-persistent) temporal correlations in the original velocity field.
    - **Checks:** logical dependency on prior mathematical claim
    - **Verdict:** UNCERTAIN; confidence: medium; impact: critical
    - **Assumptions/inputs:** Surrogate is an appropriate null model differing from the original only by removal of temporal correlations
    - **Notes:** Because the surrogate construction is incorrectly characterized as correlation-free/memoryless (and also claimed to preserve the exact marginal PDF), the stated causal attribution is not verifiable from the provided analytic argument. A corrected surrogate interpretation or alternative null model is needed to support the inference as stated.
11. ⚠ **Tail exponent condition for finite variance** (Sec. 3.4, p.5)
    - **Claim:** States that tail exponent $\beta > 3$ for $\delta v(\tau)$ implies finite variance (and thus not L\'evy-flight-like).
    - **Checks:** moment existence condition, notation/convention check
    - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
    - **Assumptions/inputs:** β is the exponent of the PDF tail: $p(|\delta v|) \sim |\delta v|^{-\beta}$ at large $|\delta v|$
    - **Notes:** Under the common convention $p(x) \sim x^{-\beta}$, variance is finite if $\beta > 3$, so the statement would be correct. The paper does not define whether β refers to the PDF tail or to the CCDF tail; without that, the implication cannot be confirmed from the PDF text alone.
12. ⚠ **CTRW trapping-time exponent criterion** (Sec. 3.4, p.5)
    - **Claim:** States that CTRW requires a power-law trapping-time distribution with exponent $\mu < 2$.
    - **Checks:** definition/convention consistency
    - **Verdict:** UNCERTAIN; confidence: low; impact: minor
    - **Assumptions/inputs:** A specific tail form for the trapping-time distribution is intended
    - **Notes:** The condition depends on the exact definition of μ and whether it refers to the PDF tail exponent or another parameterization. The paper does not specify the functional form, so the criterion cannot be audited precisely.
13. ⚠ **Velocity autocorrelation referenced but not defined** (Sec. 3.3, p.5; Fig. 3 caption p.6)

  - **Claim:** Uses normalized Lagrangian velocity autocorrelation $R_v(\tau)$ to argue for long memory.
  - **Checks:** definition completeness, notation consistency
  - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
  - **Assumptions/inputs:** $R_v(0) = 1$ normalization intended
  - **Notes:** No explicit formula is provided (vector vs component, dot product, normalization, averaging over particles and time origins). This prevents verifying consistency with the surrogate discussion and other definitions.

Limitations

- Audit is based only on the provided PDF text/images (7 pages) and contains no verification of numerical values, plots, or simulation outputs.
- Several key statistical objects (exact definitions of s^2, ω, $R_v(\tau)$, tail-exponent convention, trapping-time tail convention) are not fully specified, limiting verifiability of some analytic implications.
- The surrogate-data construction lacks implementation-level mathematical details needed for full verification (e.g., handling of conjugate symmetry for real signals), so parts are marked UNCERTAIN.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

10 items passed basic internal arithmetic/logic checks (positivity/order, exponent relationships, repeated-number equality for the provided occurrences, and simple difference/inequality computations). 1 item (CTRW trapping-time exponent criterion) remains uncertain due to lack of any μ estimate and inability to confirm whether the threshold is stated consistently across the document.

Checked items

1. ✔ **C1** (p.2, Sec. 2.1 (Numerical simulation and dataset))
   - **Claim:** Primary dataset: $N = 8000$ passive tracer particles, tracked for a total duration of 600 time units; 15 Eulerian vorticity snapshots.
   - **Checks:** integer/positive-value sanity + repeated-constant consistency
   - **Verdict:** PASS
   - **Notes:** Positivity/integer sanity passed for $N = 8000$ and 15 snapshots; duration=600 is positive. Repeated-mention consistency beyond the provided statement could not be verified.
2. ✔ **C2** (p.2, Sec. 2.1)
   - **Claim:** Kinematic viscosity is $\nu = 0.002$; forcing wavenumbers $k \in [3, 6]$.

  - **Checks:** range/order sanity
  - **Verdict:** PASS
  - **Notes:** Logical checks passed: $\nu > 0$ and $k_{\min} \leq k_{\max}$ $(3 \leq 6)$.

3. ✔ **C3** (p.2, Sec. 2.2; p.4 Fig.1 caption; p.5 Fig.2 caption)
   - **Claim:** Strain-dominated tracers: $> 70\%$ time in strain $(Q > 0)$. Vortex-dominated: $< 30\%$ time in strain.
   - **Checks:** threshold-complement gap/overlap check
   - **Verdict:** PASS
   - **Notes:** Numeric ordering holds $(30 < 70)$, implying an unclassified band of 30--70%. Could not scan for any contradictory 'exhaustive partition' claim beyond the provided statements.

4. ✔ **C4** (p.3, Sec. 2.3)
   - **Claim:** Running Hurst exponent computed via sliding window of width $\Delta t = 100$ time units; late-time focus $t > 400$.
   - **Checks:** window-feasibility with total duration
   - **Verdict:** PASS
   - **Notes:** Feasibility check passed: late-time span $= 600 - 400 = 200$, which is $\geq$ window width 100.

5. ✔ **C5** (p.1 Introduction; p.3 Sec. 2.3)
   - **Claim:** MSD scaling: $\langle \Delta x^2(t) \rangle \sim t^{2H}$; normal diffusion corresponds to $H = 0.5$; ballistic corresponds to $H = 1$.
   - **Checks:** formula-based exponent consistency
   - **Verdict:** PASS
   - **Notes:** Arithmetic under $MSD \sim t^{2H}$ is consistent: $H = 0.5 \Rightarrow$ exponent 1; $H = 1 \Rightarrow$ exponent 2. Matching to descriptive wording (e.g., 'linear/quadratic') could not be verified.

6. ✔ **C6** (p.3 Sec. 3.1)
   - **Claim:** At short times $(\tau \leq 10)$, MSD grows quadratically $(MSD \sim \tau^2)$, characteristic of ballistic motion.
   - **Checks:** ballistic mapping via MSD exponent
   - **Verdict:** PASS
   - **Notes:** Quadratic MSD exponent 2 implies $H = 2/2 = 1$, consistent with ballistic motion.

7. ✔ **C7** (p.3 Sec. 3.1; p.5 Fig.2 caption)
   - **Claim:** Late-time H for full ensemble: $H = 0.565$ (near-constant for $t > 400$).

    - **Checks:** repeated-number consistency across sections/captions
    - **Verdict:** PASS
    - **Notes:** The two provided occurrences match exactly (0.565); other potential occurrences were not available for cross-check.

8. ✔ **C8** (p.4 Sec. 3.2; p.5 Fig.2 caption)
    - **Claim:** Late-time Hurst exponents: $H_{\text{strain}} = 0.557$ and $H_{\text{vortex}} = 0.574$; claimed 'statistically identical'.
    - **Checks:** numeric difference computation (cheap)
    - **Verdict:** PASS
    - **Notes:** Pairwise absolute differences are small: $|0.574 - 0.557| = 0.017$; $|0.565 - 0.557| = 0.008$; $|0.565 - 0.574| = 0.009$, all < 0.05 (no uncertainties were provided to assess 'statistical' identity).
9. ✔ **C9** (p.4 Fig.1 caption; p.5 Sec. 3.3)
    - **Claim:** Surrogate maintains near-ballistic exponent: $H_{\text{surrogate}} \simeq 0.98$.
    - **Checks:** ballistic proximity check
    - **Verdict:** PASS
    - **Notes:** Distance to ballistic $H = 1$ is 0.02, within the provided near-ballistic threshold.
10. ✔ **C10** (p.5 Sec. 3.4)
    - **Claim:** Tail exponent $\beta > 3$ for all measured lag times; for full ensemble increases from $\beta \approx 3.36$ at $\tau = 10$ to $\beta \approx 4.84$ at $\tau = 500$.
    - **Checks:** inequality + monotonic change between two points
    - **Verdict:** PASS
    - **Notes:** Both stated points satisfy $\beta > 3$ (3.36, 4.84) and increase with lag: $\Delta\beta = 4.84 - 3.36 = 1.48$.
11. ⚠ **C11** (p.5 Sec. 3.4)
    - **Claim:** CTRW would require trapping-time exponent $\mu < 2$ (claimed not observed).
    - **Checks:** threshold sanity (definition consistency)
    - **Verdict:** UNCERTAIN
    - **Notes:** Only the criterion $\mu < 2$ is stated; no μ estimate is provided, and no verification of consistent threshold usage across the document could be performed.

Limitations

- Only parsed text from the provided 7-page PDF was available; no underlying numerical datasets (trajectories/MSD/autocorrelation arrays) were provided, limiting checks to internal arithmetic and cross-reference consistency.

- Figures are present as images; numeric verification that depends on reading plotted values or pixel-based extraction is excluded by the user’s constraints.
- Several claims use qualitative descriptors (e.g., 'nearly indistinguishable', 'slow decay', 'statistically identical') without uncertainties, confidence intervals, or sample counts for sub-populations, limiting rigorous numerical validation.
- $H(t)$ regression-based results (including late-time $H = 0.565$) could not be recomputed without the underlying MSD/trajectory data.
- Figure-based claims and distribution-shape assertions (e.g., bimodality) could not be numerically checked without extracting values from plots or having the underlying histogram/curve data.
- Statements about surrogate preservation of PSD/velocity PDF/variance require original and surrogate time-series data, which were not available for numerical verification.
- The claim that $\beta > 3$ holds for all measured lag times and all sub-populations could not be verified because only two (β, τ) points for the full ensemble are provided.
- Interpolation consistency for $Q(x,t)$ between 15 snapshots could not be checked because the time-step size and snapshot timing details are not provided.